
/70 pts

Mathematics V1205x, Calculus IIIS/IVA

Midterm Examination #1: February 14, 2001, 9:10–10:35 am

Name:__________________________

The following midterm has 7 problems which are each worth 10 points. Please read the exam carefully and check all your answers. Show all your work and justify your steps.

1. A ladybug is hovering at the point $(x, y, z) = (0, 1, 2)$. The temperature is given by the function $f(x, y, z) = ze^{xy}$.

A. In what direction does the temperature increase most rapidly? What is the maximum rate of increase in this direction?

B. What is the rate of increase of the temperature in the direction $\langle 1, 0, 0 \rangle$?

C. Find a direction in which the ladybug can move if it doesn't want to change temperature.

2. Suppose $f(x, y) = 2x^2 + 3y^2 - 4x - 5$.

A. Find the critical points for $f(x, y)$.

B. Use the method of Lagrange multipliers to find the absolute maximum and minimum of $f(x, y)$ on the region $x^2 + y^2 \leq 16$.

3. Evaluate the integral by reversing the order of integration: $\displaystyle\int_0^1 \int_{3y}^3 e^{x^2}\, dx dy$.

4. A. Suppose $f(x, y) = -16 + x^2 + 4y^2$. Find the region D for which $\int\int_D f(x, y)dA$ is a minimum. Why is it a minimum for this region? Hint: draw a graph.

B. Suppose $f(x, y) = 5 - x$. Let $D = \{(x, y)|0 \le y \le 3, 0 \le x \le 5\}$. Which of the following expressions is equal to $\int\int_D f(x, y)dA$?

i)

$$\lim_{m\to\infty}\lim_{n\to\infty}\sum_{i=1}^{m}\sum_{j=1}^{n}(5 - x)\frac{1}{n}\frac{1}{m}$$

ii)

$$\lim_{m\to\infty}\lim_{n\to\infty}\sum_{i=1}^{m}\sum_{j=1}^{n}(5 - \frac{i}{n})\frac{1}{n}\frac{1}{m}$$

iii)

$$\lim_{m\to\infty}\lim_{n\to\infty}\sum_{i=1}^{m}\sum_{j=1}^{n}(5 - \frac{5}{n})\frac{5}{n}\frac{3}{m}$$

iv)

$$\lim_{m\to\infty}\lim_{n\to\infty}\sum_{i=1}^{m}\sum_{j=1}^{n}(5 - \frac{5j}{n})\frac{5}{n}\frac{3}{m}$$

v)

$$\lim_{m\to\infty}\lim_{n\to\infty}\sum_{i=1}^{m}\sum_{j=1}^{n}(5 - \frac{3j}{n})\frac{5}{n}\frac{3}{m}$$

5. Find the volume of icecream which lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 1$.

6. Find the surface area of the part of the plane $3x + 2y + z = 6$ which lies in the first octant.

7. Let x be the number of bags of candy hearts that a student can eat in an hour. Let y be the number of miles that a student can run in an hour. The joint probability density function representing the continuous random variables x and y is $f(x, y) = \dfrac{1}{15}e^{-x/3}e^{-y/5}$ if $x \geq 0$ and $y \geq 0$ and $f(x, y) = 0$ otherwise.

A. Find the probability that a student can run less than one mile in an hour.

B. Find the expected value (mean value) of x.

Bonus: For positive constants a,b, and c find the surface area of the part of the plane $ax + by + z = c$ which lies in the first octant.