Mathematics V1205y, Calculus IIIS/IVA

Final: May 9, 2001, 9:00 am –12:00 noon.

The following final has 20 problems which are each worth 5 points. Please read the exam carefully and check all your answers. Show all your work. Some of the problems are harder than others, so move on if you get stuck.

Multiple Choice: No partial credit. Circle exactly one answer for each question.

1. **Name:**________________________

2. A hummingbird is hovering at the point $(2,4,2)$ and the surrounding temperature is given by the function $f(x,y,z) = \sqrt{xyz}$. Find the rate of change of the temperature as the hummingbird flies in the direction of the vector $\vec{v} = \langle 4, 2, -4 \rangle$.

a. 1/2 b. 1/4 c. 1/6 d. 1 e. 3/2

3. Icecream fills the space below the sphere $x^2 + y^2 + z^2 = a^2$ and above the cone $z = \sqrt{x^2 + y^2}$. The density of the icecream is given by the function $f(x,y,z) = z$. Find the mass of the icecream using spherical coordinates.

a. $\pi a^4/8$ b. $\pi a^2/\sqrt{2}$ c. $4\pi a^3/3$ d. $2\pi(1 - \frac{\sqrt{2}}{2})a^3/3$ e. $\pi a^4/4$

4. Which of the following equations is a parametric representation for this surface?

a. $x = 2 + \cos(u),\ y = 2 + \sin(u),\ z = u + \sin(v)$
b. $x = (2 + \sin(v))\cos(u),\ y = (2 + \sin(v))\sin(u),\ z = u + \cos(v)$
c. $x = u + \cos(v),\ y = (2 + \sin(v))\cos(u),\ z = (2 + \sin(v))\sin(u)$
d. $x = \sin(v)\cos(u),\ y = \sin(v)\sin(u),\ z = u$
e. $x = \cos(u),\ y = \sin(u),\ z = v$

5. Suppose $F(x, y, z) = x\mathbf{k}$ and S is the part of the plane $z = x$ lying over the square $0 \le x \le 1$ and $0 \le y \le 1$. Find the flux of F across S.

a. 2 b. $\sqrt{2}/2$ c. $\sqrt{3}/2$ d. 1/2 e. 1/4

6. A direction field is given below. Which of the following represents its differential equation?

a. $dy/dx = y^2 - 1$
b. $dy/dx = y - x$
c. $dy/dx = x^2 - y^2$
d. $dy/dx = y^2 - x^2$
e. $dy/dx = \sin(y - x)$

7. A fishtank contains 100 L of water with 7 kg of dissolved salt. Pure water enters the tank at a rate of 10 L/min. The solution is kept thoroughly mixed and drains from the tank at a rate of 10 L/min. How much salt is in the tank after 6 minutes?

a. $e^{-.6} + 6$ b. $7e^{-.6}$ kg c. $7e^{.6}$ kg d. $6e^{-7}$ kg e. $7e^{-.06}$ kg

8. In the power series solution $\sum_{n=0}^{\infty} a_n x^n$ of the differential equation $y'' - xy' - y = 0$, what recursion formula do the coefficients a_n satisfy for $n > 0$?

a. $a_{n+2} = a_n/(n+2)$ b. $a_{n+2} = a_n/n(n+1)$ c. $a_{n+2} = a_n/(n+1)$
d. $a_{n+1} = a_n/n(n+1)$ e. $a_{n+1} = a_n/(n+1)$

9. If $a + bi = (\sqrt{3} + i)^{11}$, find the pair (a, b).

a. $(a, b) = (\sqrt{3}^{11}, 1)$ b. $(a, b) = (\sqrt{3}, -1)$ c. $(a, b) = (22\sqrt{3}, 22)$
d. $(a, b) = (2^{11}\sqrt{3}, -2^{11})$ e. $(a, b) = (2^{10}\sqrt{3}, -2^{10})$

10. Suppose the probability that you will have a good summer is 1 and the probability that you will remember Stokes Theorem in the fall is 0. What is the probability that you will forget Stokes Theorem during a wonderful summer?

a. -3 b. 1 c. 40 d. 100000 e. Too much information is given to answer this

Free Response: Partial credit given. Show all your work.

1. Find the shortest distance from the origin to the surface $xyz^2 = 2$.

2. Evaluate $\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3+1}\,dx\,dy$.

3. Evaluate the integral $\int\int_R \sin(9x^2 + 4y^2)dA$ where R is the region in the xy-plane bounded by $9x^2 + 4y^2 = 1$.

4. Let $F(x, y) = \frac{-y}{x^2 + y^2}\mathbf{i} + \frac{x}{x^2 + y^2}\mathbf{j}$ and let C be the curve $x^2 + y^2 = 1$ oriented clockwise. Evaluate the line integral $\int_C F \cdot dr$.

5. Consider the vector field $F(x, y, z) = 2xi + 2yj + 2zk$. a) Compute curl$(F)$.
b) If C is any path from $(0, 0, 0)$ to (a_1, a_2, a_3) and $a = a_1i + a_2j + a_3k$, prove that $\int_C F \cdot dr = a \cdot a$.

6. Find the simple closed curve C which gives the maximal value of the following integral and explain why it yields the maximal value:

$$\int_C (x^5 - 6y + y^3)dx + (y^4 + 6x - x^3)dy.$$

7. Suppose $F(x,y,z) = (xe^z - 3y)\mathbf{i} + (ye^{z^2} + 2x)\mathbf{j} + (x^2y^2z^2)\mathbf{k}$ and S is the portion of the paraboloid $z = 4 - x^2 - y^2$ where $z \geq 0$. Compute $\int\int_S \text{curl(F)} \cdot dS$.

8. Let E be the region enclosed by the paraboloid $z = 2 - x^2 - y^2$ and the plane $z = 1$. Let S be the surface bounding E. Let $F(x,y,z) = \langle z\tan^{-1}(y^2), z^3\ln(x^2+1), z\rangle$. Find the flux of F across S; in other words find $\int\int_S F \cdot dS$.

9. Solve $y' + y = \sqrt{x}e^{-x}$ if $y(0) = 3$.

10. Solve the differential equation $y'' + 2y' + y = e^{-x}/x$ subject to the initial conditions $y(1) = 0$ and $y'(1) = e^{-1}$.