

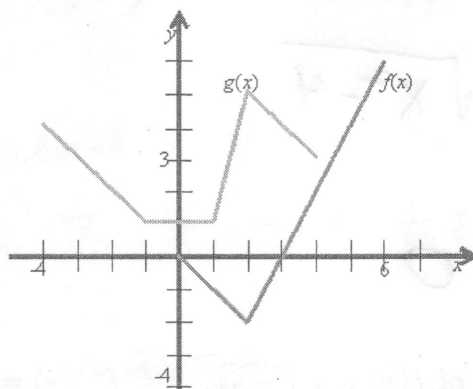
Math 123 Exam 2B

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Key



1. Evaluate each expression using the graph above.

2 (a)  $(f + g)(2) = \underline{3}$

(f) Does  $f(x)$  have an inverse for all  $x$ ? Y (N)

2 (b)  $(fg)(2) = \underline{-10}$

(g) Does  $g(x)$  have an inverse for all  $x$ ? Y (N)

2 (c)  $(g \circ f)(3) = \underline{1}$

2 (d)  $(f \circ g \circ f)(3) = \underline{-1}$

2. You want to fence off a rectangular garden adjacent to a barn (with no fence along the barn). Find the area of the largest garden possible with 100 ft of fencing.

Area = 1250



$$2x + y = 100$$

$$A(x) = (x)(100 - 2x)$$

$$= 100x - 2x^2$$

max at  $x = 25$  (so  $y = 50$ )

$$A(25) = 1250$$

3. If  $f(x) = x^2 - 7$  and  $g(x) = \sqrt{x+3}$ , find the following.

3 (a)  $f \circ g = x - 4$

3 (b)  $g \circ f = \sqrt{x^2 - 4}$

3 (c)  $g(f(2))$

$= 0$

5 4. Find the inverse of  $f(x) = \sqrt{7-4x}$ .

$y = \sqrt{7-4x}$

$x = (y^2 - 7)/(-4)$

$f^{-1}(x) = \frac{-x^2 - 7}{4} = \frac{7 - x^2}{4}$

5 5. Find the inverse of  $f(x) = \ln(x/2)$ .

$y = \ln(x/2)$

$2e^y = x$

$f^{-1}(x) = 2e^x$

6. Evaluate the following expressions.

3 (a)  $\log_2 80 - \log_2 5 = \log_2 16 = 4$

3 (b)  $\log_4 8 = 3/2$

3 (c)  $\ln \frac{e^5}{\sqrt{e}} = 9/2$

6 7. Combine into a single logarithm:  $\ln(5x) - 3\ln(x^2 + 1) + \frac{1}{2}\ln(5x - 3)$

$\ln \frac{(5x) \sqrt{5x-3}}{(x^2+1)^3}$

8. If  $\ln a = 7$ ,  $\ln b = -4$ ,  $\ln c = 8$ , evaluate the following expressions.

$$(a) \ln \frac{a^5}{b^2 c^3} = 5 \ln a - 2 \ln b - 3 \ln c$$

$$5 \cdot 7 - 2(-4) - 3(8) = 19$$

$$(b) \ln(a\sqrt{bc}) = \ln a + \frac{1}{2} \ln b + \frac{1}{2} \ln c = 9$$

$$= 7 + (-2) + (4)$$

$$(c) \ln(a/e) = \ln a - 1 = 6$$

$$7 - 1$$

9. Solve the following equations.

$$(a) 4^{x+2} = 6^{5x}$$

$$(x+2) \log 4 = 5x \log 6$$

$$x = 0.366$$

$$(b) \log_4(12 + 2x) = 3$$

$$12 + 2x = 4^3 = 64$$

$$2x = 52$$

$$x = 26$$

$$(c) 5 \ln(4 - x) = 3$$

$$\ln(4 - x) = 3/5$$

$$4 - x = e^{3/5} \Rightarrow x = 4 - e^{3/5}$$

$$x = 2.178$$

5 pts each

10. Suppose \$4,000 is invested in an account paying 6.5% interest per year (APR).

- (a) Find the amount in the account after 7 years if interest is compounded monthly.

$$6 \quad A(t) = 4000 \left(1 + \frac{0.065}{12}\right)^{7 \cdot 12} = \$6296.96$$

- (b) How long will it take for the account to have \$8,000 if interest is compounded semiannually?

$$8 \quad 4000 \left(1 + \frac{0.065}{2}\right)^{2t} = 8,000$$

$$(1.0325)^{2t} = 2$$

$$2t \log(1.0325) = \log 2$$

$$t = 10.836 \text{ yrs.}$$

- (c) Find the amount in the account after 7 years if interest is compounded continuously.

$$6 \quad A(7) = 4000 e^{(0.065)(7)} = \$6304.69$$

- (d) How long will it take for the account to have \$8,000 if interest is compounded continuously?

$$8 \quad 4,000 e^{0.065t} = 8,000$$

$$0.065t = \ln(2)$$

$$t = 10.66 \text{ yrs.}$$