

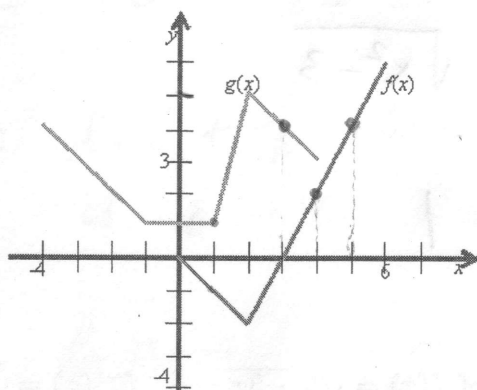
Math 123 Exam 2A

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NAME: _____

Key



10

1. Evaluate each expression using the graph above.

2 (a) $(f + g)(3) = \underline{4}$

(f) Does $f(x)$ have an inverse for all x ? Y ☒ N

2 (b) $(fg)(1) = \underline{-1}$

(g) Does $g(x)$ have an inverse for all x ? Y ☒ N

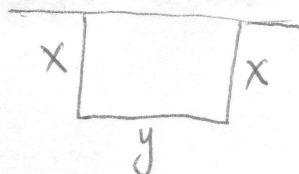
2 (c) $(g \circ f)(4) = \underline{5}$

2 (d) $(f \circ g \circ f)(4) = \underline{4}$

10

2. You want to fence off a rectangular garden adjacent to a barn (with no fence along the barn). Find the area of the largest garden possible with 80 ft of fencing.

Area = 800



$$2x + y = 80$$

$$\begin{aligned} A(x) &= (x)(80 - 2x) \\ &= 80x - 2x^2 \end{aligned}$$

Max at $x = 20$ (so $y = 40$)

$$A(20) = 800$$

3. If $f(x) = x^2 - 8$ and $g(x) = \sqrt{x+5}$, find the following.

3 (a) $f \circ g = x - 3$

3 (b) $g \circ f = \sqrt{x^2 - 3}$

3 (c) $g(f(2)) = 1$

5 4. Find the inverse of $f(x) = \sqrt{5-3x}$. $f^{-1}(x) = -\frac{x^2-5}{3} = \frac{5-x^2}{3}$

$$y = \sqrt{5-3x}$$

$$x = (y^2 - 5) / -3$$

5 5. Find the inverse of $f(x) = \ln(x/3)$. $f^{-1}(x) = 3e^x$

$$y = \ln\left(\frac{x}{3}\right)$$

$$3e^y = x$$

6. Evaluate the following expressions.

3 (a) $\log_4 80 - \log_4 5 = \log_4 16 = 2$

3 (b) $\log_8 4 = 2/3$

3 (c) $\ln \frac{e^3}{\sqrt{e}} = 5/2$

6 7. Combine into a single logarithm: $\ln(5x) + 3\ln(x^2+1) - \frac{1}{2}\ln(3x-1)$

$$\ln \frac{(5x)(x^2+1)^3}{\sqrt{3x-1}}$$

15 8. If $\ln a = 4$, $\ln b = -8$, $\ln c = 6$, evaluate the following expressions.

$$\begin{aligned} \text{(a) } \ln \frac{a^5}{b^2 c^3} &= 5 \ln a - 2 \ln b - 3 \ln c \\ &= 20 + 16 - 18 = 18 \end{aligned}$$

$$\begin{aligned} \text{(b) } \ln(a\sqrt{bc}) &= \ln a + \frac{1}{2} \ln b + \frac{1}{2} \ln c = 3 \\ &= 4 - 4 + 3 \end{aligned}$$

$$\begin{aligned} \text{(c) } \ln(a/e) &= \ln a - 1 = 3 \\ &= 4 - 1 \end{aligned}$$

15 9. Solve the following equations.

$$\text{(a) } 6^{x+2} = 4^{5x}$$

$$(x+2) \log 6 = 5x \log 4$$

$\log 6 = 0.778$ $\log 4 = 0.602$

$$x = \cancel{0.697} \quad 0.697$$

$$\text{(b) } \log_3(11 + 2x) = 4$$

$$11 + 2x = 3^4 = 81$$

$$2x = 70$$

$$x = 35$$

$$\text{(c) } 4 \ln(6 - x) = 3$$

$$\ln(6 - x) = \frac{3}{4}$$

$$6 - x = e^{3/4} \Rightarrow x = 6 - e^{3/4}$$

$$x = 3.88$$

10. Suppose \$5,000 is invested in an account paying 4.5% interest per year (APR).

(a) Find the amount in the account after 6 years if interest is compounded monthly.

6
$$A(6) = 5,000 \left(1 + \frac{0.045}{12}\right)^{6 \cdot 12} = \$6,546.52$$

(b) How long will it take for the account to have \$8,000 if interest is compounded semiannually?

8
$$5,000 \left(1 + \frac{0.045}{2}\right)^{2t} = 8000$$

$$(1.0225)^{2t} = \frac{8}{5}$$

$$2t \log(1.0225) = \log \frac{8}{5}$$

$$t = 10.56 \text{ yrs.}$$

(c) Find the amount in the account after 6 years if interest is compounded continuously.

6
$$A(6) = 5,000 e^{(0.045)(6)} = \$6549.82$$

(d) How long will it take for the account to have \$8,000 if interest is compounded continuously?

8
$$5,000 e^{(0.045)t} = 8000$$

$$(0.045)t = \ln\left(\frac{8}{5}\right)$$

$$t = 10.44 \text{ yrs.}$$