Calculus IIIS/IVA – Second Midterm Exam Prof. Ilya Kofman, Nov. 13, 2001

WRITE YOUR NAME AND MY NAME ON YOUR EXAM BOOKLET.
Show all your work. You can earn partial credit only if you justify your steps.
No calculators are permitted on this exam.

1.**(12 points. One point for a correct answer; minus one for an incorrect answer.)** In the following C is a CLOSED curve, S is a CLOSED surface, F is a vector field and f is a function. Which of the following are defined and equal zero:

1. grad curl(F)
2. curl grad(f)
3. div curl(F)
4. curl div(F)
5. grad div(F)
6. div grad(f)
7. $\int_C \text{curl}(F) \cdot dr$
8. $\int_C \text{grad}(f) \cdot dr$
9. $\int_C \text{div}(F) dr$
10. $\int\int_S \text{grad}(f) \cdot n \, dS$
11. $\int\int_S \text{curl}(F) \cdot n \, dS$
12. $\int\int_S \text{div}(F) \, dS$

2.**(8 points)** Let C be a simple closed curve in the plane. Let $F = -ydx + xdy$. Show, using one of the theorems, that $\int_C F \cdot dr$ is related to the area enclosed by C.

3.**(30 points)** Let F be the vector field $(3y, \ 0, \ 0)$. Let Σ be the lower unit hemisphere oriented with the downward normal and with boundary C:

$$\Sigma = \{(x, \ y, \ z) \mid x^2 + y^2 + z^2 = 1; \ z \leq 0\}.$$

(a) Directly compute $\int_C F \cdot dr$.

(b) By Stokes' theorem this line integral is equal to a surface integral over Σ. Directly compute this surface integral.

4.**(25 points)** Let C be one turn of the helix:

$$C = \{(\cos(t),\ \sin(t),\ t) | 0 \le t \le 2\pi\}.$$

Compute $\int_C F \cdot dr$ where $F = (ze^{xz},\ 0,\ xe^{xz})$. [Hint: F is conservative.]

5. **(25 points)** An open bottle B stands vertically on the xy-plane and its lip (or boundary) is a unit circle in a plane parallel to the xy-plane. The bottle has volume $750m\ell$. Compute

$$\int\int_B F \cdot n \, dS$$

where n is the outward unit normal and

$$F = (x + y^2,\ x(z^2 + 1),\ 1).$$

Bonus Question:

Let C be the ellipse $x^2 + 4y^2 = 4$ oriented counterclockwise. Compute

$$\int_C \left(\frac{-ydx + xdy}{x^2 + y^2} \right) + \left(\frac{-ydx + (x-1)dy}{(x-1)^2 + y^2} \right).$$