Calculus IIIS/IVA – First Midterm Exam Prof. Ilya Kofman, Oct. 4, 2001

Show all your work. You can earn partial credit only if you justify your steps. No calculators are permitted on this exam.

1. Let C be ice cream inside the cone $z = \frac{1}{2}\sqrt{x^2+y^2}$ and inside the sphere $x^2+y^2+z^2 = 9$. Suppose the density of C at each point (x, y, z) is equal to $K\sqrt{x^2+y^2+z^2}$ for a positive constant K. Compute the total mass of C.

2. Evaluate
$$\int_0^1 \int_{\sqrt[3]{y}}^1 \sqrt{x^4+1}\, dx\, dy$$

3. Let a, b and c be positive constants. Let P be the plane given by $ax+by+cz = 2$. What is the surface area of the part of P contained in the first octant ($x \geq 0$, $y \geq 0$, $z \geq 0$)?

4. Compute the volume inside the cylinder $x^2+y^2 = 2y$ above the xy-plane and below the plane $z = x+1$.

5. Evaluate $\int\int_Q x^2 - y^2\, dx\, dy$ by changing variables, where Q is the region in the plane bounded by the lines $x-y = 0$, $x-y = 1$, $x+y = -2$ and $x+y = 2$.

Bonus Question (optional): Rewrite the following integral in the order $dx\, dy\, dz$.
$$\int_0^1 \int_{1-x}^1 \int_x^1 f(x, y, z)\, dz\, dy\, dx$$

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