MINERVA RESEARCH FOUNDATION

LECTURES

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SPRING 2009

Professor **Hans FOELLMER**

Humboldt University, Berlin

Convex Risk Measures and Their Dynamics

Lectures at Columbia University, February - April 2009 *Room 1025*
School of Social Work, 1255 Amsterdam Ave.

Tuesdays and Thursdays, 10:00 – 12:00
First Lecture: Tuesday February 17th

Abstract: We discuss recent advances in the mathematical quantification of financial risk. The standard approach in terms of Value at Risk has serious deficiencies. This has motivated a systematic analysis of risk measures which satisfy some minimal requirements of coherence and consistency. Our focus will be on the basic structure theorems for convex risk measures, on the role of law-invariance, and on the dynamics of risk measures as new information comes in. We shall also describe the connections with recent developments in the microeconomic theory of preferences in the face of uncertainty and model ambiguity.

SPRING 2009

Professor **Sara BIAGINI**

University of Pisa

Topics in Portfolio Optimization with general underlying Assets

Lectures at Columbia University, January 2009 *Room 1025 SSW*

Lecture 1: Tuesday Jan 20, 9:30AM-11AM
Lecture 2: Wednesday Jan 21, 9:30AM-11AM
Lecture 3: Thursday Jan 22, 9:30AM-11AM
Lecture 4: Friday Jan 23, 9:30AM-11AM
Lecture 5: Tuesday Jan 27, 9:30AM-11AM

Aim of the Lectures and Prerequisites: We provide a theoretical framework for portfolio optimization with general, possibly non-locally bounded, processes. Some familiarity is assumed with: 1) the basic concepts of stochastic calculus, such as predicable processes, (super, local) martingales, semimartingales and stochastic integration; 2) functional and convex analysis, especially for Lecture 5. However we will try to be as self-contained as possible.

Lecture 1: The market model and absence of arbitrage. The increasing complexity of financial instruments requires more general models for the underlying assets, which can be non-locally bounded. We give some examples, including Levy models. In such generality, the notion of No Arbitrage must be replaced by No Free Lunch with Vanishing Risk. In the Fundamental Theorem of Asset Pricing for non-locally bounded processes, Delbaen and Schachermayer 1998 showed that the pricing measures in this context are the *sigma*-martingale measures instead of (local) martingale measures. The mathematical concept of *sigma*-martingale process was introduced by Chou and Emery in the '70s and it is a generalization of the martingale concept. We illustrate it in a variety of examples.

Lecture 2: Which set of admissible strategies? The choice of a good set of admissible strategies is a fundamental and highly non-trivial issue. Harrison and Kreps noted that a certain type of strategy, "the doubling" strategy, starting with zero money generates a positive net return with probability one and within a finite time. Such strategies violate with their inconsistency the foundations of Mathematical Finance and the No-Arbitrage Pricing Theory. Therefore, since Harrison and Kreps a wide variety of constraints has been proposed in order to rule out the doubling strategy. The class of strategies widely used in the applications, like portfolio selection, are the uniformly bounded-from-below strategies H which have nice mathematical properties (an application of the Ansel-Stricker Lemma gives that these processes are local martingales and supermartingales) and a clear financial interpretation (finite credit line during the trading). But if one wants to account for unbounded stock prices, the set H is not enough, as it may reduce to the trivial zero strategy: $H = \{0\}$. There have been so far some proposals (e.g. Delbaen and Schachermayer 1998 in the super-replication price problem, Biagini and Frittelli for utility maximization) in order to define a good set of strategies in such a way to account for general asset prices and still preserve the features of the Ansel-Stricker lemma. We focus on the Biagini-Frittelli definition of admissible set H^w, consisting of strategies which are bounded from below by a *random* control W.

Lecture 3: Utility Maximization (A). The set of strategies Hw performs well in applications, the case study here analyzed being expected utility maximization from terminal wealth. After giving some precise mathematical definitions, we point out how the problems of maximizing utility with restrictions on the wealth (say, the utility $U(x) = \log x$) and of maximizing unrestricted utility (say $U(x) = 1-e^{-x}$) can be unified by the use of Orlicz spaces. We will see that these spaces, generalizations of the classic L_p spaces, are naturally induced by the utility function U itself and thus provide a natural framework for the problem.

Lecture 4: Utility Maximization (B). We go into the details of the proofs of the optimization problem, solved via duality methods. The dual problem has the nice feature of being defined over the *sigma*-martingale measures. Also, the optimal investment H^* satisfies some nice properties. We show how these results can be extended to cover the problem of utility maximization with *random endowment*.

Lecture 5: The indifference price as a risk measure. This new Orlicz formulation enables several key properties of the indiference price $p(B)$ of a claim B satisfying conditions weaker than those assumed in the current literature. In particular, the indiference price functional turns out to be, apart from a sign, a convex risk measure on the Orlicz space induced by the utility function U.

We conclude the lectures by pointing out some open problems.

FALL 2008

Professor **Shige PENG**

Institute of Mathematics
Sandong University, China

Coherent and Convex Risk Measures, and the Related Time-Consistent Nonlinear Expectations

Lectures at Columbia University, October-November 2008

Abstract: In this series of lectures we present some recent research results on discrete and continuous-time coherent risk measures in finance, under mean and variance uncertainty. We shall give a systematic analysis on mean, variance and dependence uncertainties. For discrete-time situations, we consider financial products underlying a large sum of relatively small risk exposures under mean and variance uncertainty, as well as dependence uncertainty. The notion of *sublinear expectation* (also called coherent risk measure, upper expectation, coherent prevision), of which the well-known relation *E[X] + E[Y] = E[X + Y]* in classical probability theory becomes the following sublinear one *ˆE[X] + ˆE[Y] \le ˆE[X + Y]* , is shown to be a basic tool for superhedging, for superpricing, and for measures of risk. The related notions of 'robust' independence and identical distribution (iid) will lead us to derive a new type of law of large numbers and central limit theorem (LLN & CLT), under a space of sublinear expectation *(\Omega, F, ^E)* instead of the well-known framework of probability space *(\Omega, F, P),* with a limiting distribution that replaces the classical normal N(μ, s^2). This new distribution is stable under *ˆE* and is called *G-normal distribution*.

This is also the starting point a new theory of random processes and stochastic calculus, which gives us a new insight to characterize and calculate varies kinds of financial risk. A G-Brownian motion *B(t), t \ge 0,* is a path-wise continuous stochastic process defined in a sublinear expectation space *(\Omega, F, ^E)* with stationary and independent increments. We present the related random and stochastic calculus, as well as their applications to option pricing and to measures of risk in finance, under drift and volatility uncertainty, and discuss the related data analysis. A path-interpretation for the related fully nonlinear PDE also plays an important role in our lectures.

A broad survey will be given on:
• Nonlinear expectation, nonlinear distributions and the related notion of independence;
• Backward stochastic differential equations and the related g-expectations and g-risk measures;
• Stochastic Hamiltonian system and stochastic PDE
• G-Brownian motion and related stochastic analysis of Itˆo's type;
• Quasi-linear and fully nonlinear PDEs and the corresponding path interpretations.

SPRING 2008

Professor **Michel EMERY** (University of Strasbourg)

"Five Lectures on Manifold-Valued Semimartingales"

Tentative Times and Location

Conference Room 1025 SSW, 1:00 – 2:30 Thursday April 10^{th}, 17^{th}

Conference Room 1025 SSW, 9:00 – 10:30 Thursday April 24^{th}

Conference Room 1025 SSW, 1:00 – 2:30 Tuesday April 15^{th}, 22^{nd}

Prerequisites: Some familiarity with the basic definitions of continuous stochastic calculus will be assumed: previsible process, continuous martingale, continuous local martingale, continuous semimartingale, stochastic integration. No knowledge of differential geometry will be needed: the basic notions will be recalled, and connections, the only geometric tool that will be used, will appear only at the end.

Lecture 1: The notion of a *formal semimartingale* will be introduced and explained. The space of formal semimartingales contains the space of semimartingales; if S is a semimartingale and H a previsible process, the stochastic integral $\int H dS$ can always be defined as a formal semimartingale. This possibility of performing stochastic calculus with no integrability constraints on H gives much freedom, just as distributions liberate ordinary calculus from differentiability constraints. (By the way, the inventor of formal semimartingales is the same Laurent Schwartz whose name is associated to distributions.) Interesting fallouts of this notion: three definitions are made much simpler and more tractable, that of the space $L(S)$ of all previsible processes integrable with respect to a given semimartingale S, that of stochastic integration of vector-valued processes, and that of a σ-martingale.

We shall also recall some basic notions from (ordinary) differential geometry: differentiable manifolds, tangent and cotangent spaces and bundles.

Lecture 2: *Second order differential geometry* will be presented. It was devised by L.Schwartz to describe intrinsically the infinitesimal behavior of manifold-valued semimartingales, but can be understood in a non-probabilistic framework. The tangent space to a manifold at a point x consists of the speeds of all possible curves passing at x; similarly, the second-order counterpart to the tangent space contains the accelerations of all those curves. Second-order tangent and cotangent spaces and bundles will be introduced, as well as Schwartz morphisms (the natural morphisms between second-order tangent spaces). Purely geometric as they are, these objects will not be found in any textbook on differential geometry.

Then *manifold-valued semimartingales* are introduced, at last! Schwartz' principle says that if X is such a process, then dX (which, as everyone knows, is a convenient notation but does not exist) belongs to the realm of second-order geometry.

Lecture 3: Schwartz' principle will be illustrated by second-order stochastic integration, that is, integration of second-order covectors against a (manifold-valued) semimartingale. We shall then turn to a general (hence useful) theory of intrinsic second-order stochastic differential equations in manifolds. This concept will be made clearer by analogy with ordinary differential equations between manifolds: given two manifolds M and N, an ODE is a machine that transforms M-valued curves into N-valued ones, and similarly a SDE transforms M-valued semimartingales into N-valued ones.

By far, the most important examples and applications of this theory belong to *Stratonovich intrinsic stochastic calculus*, which was already known and practiced in the sixties, long before Schwartz pondered over second-order calculus around 1980. We shall see how purely geometric operations can transform ordinary 1-forms or ODEs into second-order forms or SDEs, giving rise to the ubiquitous Stratonovich transfer principle: geometric constructions on curves extend canonically to semimartingales.

Lecture 4: We shall elaborate on Stratonovich calculus considered as a particular case of Schwartz' calculus, and derive the theory of Stratonovich SDEs from the theory of second-order SDEs as presented in the previous lecture. We shall also describe a general approximation scheme by time-discretization, which yields Stratonovich objects.

A very important example consists of stochastic lifts and stochastic transport of vectors or tensors, of fundamental importance in practice. This will need the definition of a *connection*, a most fundamental tool in differential geometry.

Lecture 5: Much less well known than Stratonovich calculus, an *Ito intrinsic stochastic calculus* is also possible in manifolds. It requires some additional geometric structure, namely, a connection, but needs less smoothness than Stratonovich calculus. We shall introduce Ito stochastic integrals, Ito stochastic differential equations, the \Ito transfer principle, and an approximation scheme of Ito objects by time-discretization.

A few words will be said about manifold-valued martingales.

Lecture notes are available at **http://hal.archives-ouvertes.fr/hal-00145073/fr/**

MINERVA RESEARCH FOUNDATION LECTURES

FALL 2007

Professor **Alison ETHERIDGE** (Oxford University)

"Some mathematical models from population genetics"

Tentative Times and Location:

Conference Room 1025 SSW

Wednesday	5 Sept	4:30 - 5:45
Thursday	6 Sept	4:30 - 5:45
Friday	7 Sept	2:00 - 3:15
Monday	10 Sept	4:30 - 5:45
Thursday	13 Sept	4:30 - 5:45

Lecture 1: Classical models

We introduce some terminology and review some established models from mathematical population genetics. This includes Wright-Fisher, Moran and stepping stone models for the (forwards in time) evolution of gene frequencies and Kingman's coalescent and its extensions for modeling the genealogical relationship (found by tracing backwards in time) between individuals in a sample from a population.

Lecture 2: Recombination

In sexually reproducing organisms such as our own, in which chromosomes are carried in pairs, each individual will inherit one chromosome of each pair from their mother and one from their father. But their chromosomes are not faithful copies of parental chromosomes. One reason for this is recombination. We describe the action of recombination and explain the resulting mathematical complications in our models. Backwards in time analytic results are hard to find. Here we consider the simpler problem of the descent of a block of genome forwards in time. Our model, based on branching processes, predicts the probability of survival of any genetic material from a single block of genome \$t\$ generations in the future.

Lecture 3: Selection

If a selectively advantageous mutation appears in a population, then with some probability it will increase in frequency until everyone in the population carries it. We then say that a selective sweep has occurred.How can we detect selective sweeps in data? The key is an effect known as genetic hitchhiking which we discuss here. We then turn to another form of selection, balancing selection, which contrives to maintain different forms of the same gene at non-trivial frequencies in the population. Examination of the effect on gene frequencies at a neutral locus on the same chromosome has important ramifications for the classical models introduced in the first lecture.

Lecture 4: Spatial models

An important conclusion from the work on selection described in lecture 3 is that we cannot disentangle the effects of demography and genetics. In this lecture we describe some of the challenges of demographic modeling and investigate which features of a demographic model must be understood if we are to feed demographic information into our genetic models.

Lecture 5: Muller's ratchet and the rate of adaptation

The evolutionary force of recombination is lacking in an asexually reproducing population. As a consequence, the population can suffer an irreversible accumulation of deleterious mutations, a phenomenon known as Muller's ratchet. Since recombination can overcome this effect, it is sometimes used as an explanation for the evolution of sex. But other forces can overcome the ratchet too; for example the presence of some beneficial mutations. A mathematical model for the ratchet was formulated by Haigh (1978), but in spite of the apparent simplicity of the formulation
the model has proved to be remarkably resistant to analytic study. We investigate variants of Haigh's model in two settings: first when there are only deleterious mutations and second when a small proportion of mutations are beneficial. In the latter case we discover that large enough populations increase in mean fitness and we establish a lower bound on this 'rate of adaptation'.

Lecture notes are available at the site **http://www.stats.ox.ac.uk/~etheridg**

MINERVA RESEARCH FOUNDATION LECTURES

Spring 2007

Professor **Willliam SUDDERTH** (University of Minnesota)

Stochastic Dynamic Programming

Wednesday *2 May*, Monday *7 May*, Wednesday *9 May*, Friday *11 May*: 10:30-12:00 AM
10th Floor Conference Room, SSW Bldg, Department of Statistics

Synopsis

Stochastic dynamic programming is the theory of how to "control" a stochastic process so as to maximize the chance, or the expected value, of some objective. For example, a player might try to choose bets (investments) to maximize the chance of winning $1000 while playing roulette (the stock market). The subject is also known as stochastic control, Markov decision theory, or even gambling theory. The multitude of names is due in part to the many fields of application, which include statistics, economics, operations research, and mathematical finance. The lectures will cover the basic theory of discrete-time dynamic programming including, as time permits, backward induction, discounted dynamic programming, positive, and negative dynamic programming. Several examples will be treated in detail.

Prerequisites: Some knowledge of Probability Theory.

Some References: R. Bellman (1957) Dynamic Programming.
D. Bertsekas & S.E. Shreve (1978) Stochastic Optimal Control.
S. Ross (1983) Stochastic Dynamic Programming.
N. Stokey & R. Lucas (1989) Recursive Methods in Economic Dynamics.
M. Puterman (1994) Markov Decision Processes.
A. Maitra & W. Sudderth (1996) Discrete Gambling and Stochastic Games.