PROBABILITY SEMINAR

The Probability Seminar takes place ***Fridays from 11:00 to 12:00 noon***. Unless indicated otherwise, talks in Spring 2008 are held in

Room 903, 9th Floor in the School of Social Work (Department of Statistics), 1255 Amsterdam Avenue between 121st - 122nd streets.

Half an hour before each talk, we usually meet for coffee and tea in the Statistics Departmental Lounge on the 10th floor of that building.

For information on how to get to Columbia University and find the Department of Statistics, please check our campus map at http://www.columbia.edu/about_columbia/map/

SPECIAL MATHEMATICAL FINANCE SEMINAR

Tuesday, January 22nd, 1:10-2:25 in Rm. 307 Math

Professor Alexander CHERNY, Moscow State University & Bloomberg

Combining Factor Risks in Risk Measurement Schemes

Abstract: This paper is related to the measurement of the risk of a portfolio, the risk being driven by multiple factors. The problem under consideration is proper combining of the risks brought by individual factors, with a view of dependence between them. The practical importance of the problem stems from the fact that one can effectively estimate the joint law of a portfolio and each single factor but, due to the lack of data, cannot estimate the joint law of portfolio and all factors. We propose a methodology based on: the notion of factor risks independently introduced in the industry by the RiskData and in the literature by Cherny and Madan; selecting a minimal multifactor risk profile matching single-factor risk profiles. Within this approach we provide a solution in the model, where the distribution of risk factors is a Gaussian copula. The solution is computationally feasible as it is reduced to inverting a matrix of a reasonable dimension.
(Joint work with Raphael Douady, Risk Data and Stanislav Molchanov, UNC Charlotte.)

Joint Probability-Topology Seminar

In Room 520 Mathematics

. Friday January 25: Prof. **Richard KENYON** (Brown)

The configuration space of branched polymers

In this talk a "branched polymer" will be a connected collection of unit disks with non-overlapping interiors. Building on and from the work of Brydges and Imbrie, we give an elementary calculation of the volume of the space of branched polymers with n disks in the plane and in 3-space. Our development reveals some more general identities, and allows exact random sampling. In particular we show that a random 3-dimensional branched polymer with n disks has diameter of order $\sqrt{n}$. *(Joint work with Peter Winkler.)*

. Friday February 8: Prof. **Robin PEMANTLE** (U. Penn)

Asymptotics of ensembles with multivariate rational generating functions

Abstract: We consider a number of problems concerning random tilings in which there is a simple rational generating function but seemingly complicated asymptotic behavior. Examples are (1) Arctic circle phenomenon for Aztec Diamond tilings; (2) a similar phenemonon for Cube Groves; (3) the so-called Diabolo or Fortress model.

In each case, previously understood asymptotic theory does not apply, due to singularities of the pole surface. We now know how to deal with these.

In the first half of the talk I will discuss the examples, give the (simple) generating functions, and show pictures of the models and asymptotic phenomena. In the second half, I will summarize the theory of how one obtains asymptotics from the generating function. In the third half, I will show how one overcomes the technical difficulties arising from the singularities or the pole surface. *(Joint work with Yuliy Baryshnikov.)*

SPECIAL PROBABILITY – APPLIED MATHEMATICS SEMINAR

Tuesday Feb. 12th, 2:45-3:50 Rm. 214 Mudd

Professor Jonathan MATTINGLY, Duke University

"Ergodicity, Energy Transfer, and Stochastic Partial Differential Equations"

. **Friday February 15:** Prof. **Jay ROSEN** (CUNY)

Existence of a critical point for the infinite divisibility of squared Gaussian vectors

Abstract: Let (G_1,\ldots, G_n) be a mean zero Gaussian vector in R^n with covariance matrix \Gamma. Necessary and sufficient conditions for the infinite divisibility of (G^2_1,\ldots, G^2_n) in terms of \Gamma have been known for some time. More recent results of Eisenbaum & Kaspi give necessary and sufficient conditions in terms of \Gamma for ((G_1+r)^2,\ldots, (G_n+r)^2) to be the infinitely divisible for all real r. This is related to the local times of an associated Markov process. We show that for certain \Gamma there exists a critical r_0>0 such that ((G_1+r)^2,\ldots, (G_n+r)^2) is infinitely divisible for all |r| \leq r_0 but not for |r|>r_0. *(Joint work with Michael Marcus.)*

SPECIAL MATHEMATICAL FINANCE SEMINAR

Thursday, February 21st, Room 1025 SSW Bldg
1:10-2:25 PM

Professor Alexander SCHIED, Cornell University

Analysis of a Control Problem arising in Optimal Portfolio Liquidation

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. Friday February 22: Dr. Gerardo HERNANDEZ (Columbia)

On Schr\"odinger's equation, 3-dimensional Bessel bridges, and passage time problems

Abstract: The aim of this talk is finding an explicit representation of the density *\varphi_f* of the first time *T* that a one-dimensional Brownian process *B(.)* reaches the moving boundary *f(.),* where *f(t):=a+\int_0^t f'(s)ds* and *T:=inf{t\geq 0|B_t=f(t)},* given that *f''(t)>0*, for all *t\geq 0*. We do so, by first finding the expected value of the following functional

\mathbb{E}\left[\exp\left\{-\int_0^sf''(u)\tilde{X}_u du \right\}\right],

of the 3-dimesional Bessel bridge process \tilde{X} [the reader may consult for instance Chapter 11 in Revuz & Yor (2005) for a general overview of this process], then exploiting its relationship with first-passage time problems as pointed out by Kardaras (2007). It turns out that this problem is related to Schr\"odinger's equation with time-dependent linear potential, see Feng (2001). As a by-product we solve for a family of Volterra integral equations, which were previously only treated numerically, see Peskir (2001).

Feng, M. (2001). Complete solution of the Schr\"odinger equation for the time-dependent linear potential . *Phys. Review* A **64**, 034101.
Karatzas, I. and S. Shreve. (1991). *Brownian Motion and Stochastic Calculus*, Springer-Verlag.
Kardaras, K (2007). On the density of first passage times for difussions. *In preparation*.
Martin-L\"of. (1998). The final size of a nearly critical epidemic, and the first passage time of a Wiener process to a parabolic barrier. *J. Appl. Prob.* **35**.
Peskir, G. (2001). On integral equations arising in the first-passage problem for Brownian motion, *J. Integral Equations Appl.* **14**.
Revuz, D., and M. Yor. (2005). *Continuous Martingales and Brownian Motion*, Springer-Verlag.

. Friday February 29: Prof. Isaac MEIJLISON (Tel Aviv)

On the adjustment coefficient, drawdowns, and Lundberg-type bounds for random walk

Abstract: Consider a random walk whose (light-tailed) increments have positive mean. Lower and upper bounds are provided for the expected maximal value of the random walk until it experiences a given drawdown d. These bounds, related to the Calmar ratio in Finance, are of the form (exp{alpha d}-1)/alpha and (K exp{alpha d}-1)/alpha for some K>1, in terms of the adjustment coefficient alpha (defined by E[exp{-\alpha X}]=1) of the insurance risk literature. Its inverse 1/alpha has been recently derived by Aumann & Serrano as an index of riskiness of the random variable X. This article also complements the Lundberg exponential stochastic upper bound and the Cramer-Lundberg approximation for the expected minimum of the random walk, with an exponential stochastic lower bound. The tail probability bounds are of the form C exp{-alpha x} and exp{-alpha x} respectively, for some (1 \over K) < C < 1.

Our treatment of the problem involves Skorokhod embeddings of random walks in Martingales, especially via the Azema--Yor and Dubins stopping times, adapted from standard Brownian Motion to exponential Martingales.

. **Friday March 7:** Dr. **Vasileios MAROULAS** (Chapel Hill)

Small noise large deviations for infinite dimensional stochastic dynamical systems

Abstract: Freidlin-Wentzell theory, one of the classical areas in large deviations, deals with path probability asymptotics for small noise stochastic dynamical systems. For finite dimensional stochastic differential equations (SDE) there has been an extensive study of this problem. In this work we are interested in infinite dimensional models, i.e. the setting where the driving Brownian motion is infinite dimensional. In recent years there has been lot of work on the study of large deviations principle (LDP) for small noise infinite dimensional SDEs, much of which is based on the ideas of Azencott (1980). A key in this approach is obtaining suitable exponential tightness and continuity estimates for certain approximations of the stochastic processes. This becomes particularly hard in infinite dimensional setting where such estimates are needed with metrics on exotic function spaces (e.g. Hölder spaces, spaces of diffeomorphisms etc).

Our approach to the large deviation analysis is quite different and is based on certain variational representation for infinite dimensional Brownian motions. It bypasses all discretizations and finite dimensional approximations and thus no exponential probability estimates are needed. Proofs of LDP are reduced to demonstrating basic qualitative properties (existence, uniqueness and tightness) of certain perturbations of the original process. The approach has now been adopted by several authors in recent works to study various infinite dimensional models such as stochastic Navier-Stokes equations, stochastic flows of diffeomorphisms, SPDEs with random boundary conditions.

As a first example of this approach, we consider a class of stochastic reaction-diffusion equations, which have been studied by various authors. We establish a large deviation principle under conditions that are substantially weaker than those available in the literature. We next study a family of stochastic flows of diffeomorphisms that arise in certain image analysis problems. Large

deviations for the case where the driving noise is finite dimensional has been studied by Ben Arous and Castell (1995). We extend these results to an infinite dimensional setting and apply them to a problem of image analysis.

Professor Peter FRIZ

Cambridge University

Mini-Course on Stochastic Analysis via Rough Paths

Tue-Thu March 25-27
Tue April 1

Room 1025 SSW Bldg, 1:00 – 2:20

Ordinary differential equations of form $dy\left(t\right) =V\left(y\right) \cdot dx\left(t\right) $, where $V=\left(V_{1},\dots ,V_{d}\right) $ is a collection of vector fields and x a d-dimensional input signal, arise naturally in various parts of pure and applied mathematics. In essence, T. Lyons' *Rough Path Analysis* is a collection of highly non-trivial estimates for such equations.

The construction of diffusion processes led K. It\^{o}'s to take $x\left(t\right) =B\left(t,\omega \right) $, a d-dimensional Brownian sample path (hence of unbounded variation), and subsequently to his martingale-based theory of stochastic differential equations. The resulting solution map, known as It\^{o}-map, $B\left(\cdot ,\omega \right) \mapsto y\left(\cdot ,\omega \right) $ is notorious for its lack of continuity and this is precisely the difficulty in proving key theorems in diffusion theory, such as sample path large deviations or support theorems for $y\left(\cdot ,\omega \right) $.

Rough path theory on the other hand explains how the ODE solution map $x\in C^{1}\mapsto y\in C^{1}$ can be continuously extended to the closure in various rough path metrics so that $x\left(t\right) =B\left(t,\omega \right) $ can be accommodated after all. There is a conceptual price to pay: x has to be replaced by a path with values in a (free nilpotent) Lie group. In the case of Brownian motion this amount to replace B by the *Enhanced Brownian motion* $\mathbf{B}=\left(B,A\right) $ where A is L\'{e}vy's stochastic area. The It\^{o} map then factorizes to

$$
B\left(\cdot ,\omega \right) \mapsto \mathbf{B}\left(\cdot ,\omega \right)
\mapsto y\left(\cdot ,\omega \right)
$$

where $B\mapsto \mathbf{B}$ is only measurable while the mapping $\mathbf{B} \mapsto y$ is deterministic and continuous business. As a consequence, many properties of $y\left(\cdot

,\omega \right) $ reduce automatically to properties of $\mathbf{B}\left(\cdot ,\omega \right) $.

After a reasonably self-contained working introduction to rough path theory, I shall center this mini-course around Enhanced Brownian motion and the resulting corollaries for stochastic differential equations. Topics which I intend to cover include:

. Large deviations for $\mathbf{B}$~and Freidlin-Wentzell theory.
. Support description for $\mathbf{B}$ and the Stroock-Varadhan support theorem.
. Limit Theorems for Stochastic Flows.
. Regularity of $\mathbf{B}\mapsto y$ beyond Malliavin and non-degeneracy of solutions to rough differential equations.

The lectures will be accessible to graduate students in probability. A preliminary account can be found here

http://www.statslab.cam.ac.uk/~peter/Columbia2008/roughpaths.htm

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SPECIAL PROBABILITY SEMINAR

Wednesday March 26th, 3:00-4:00 (TBA)

Professor Julien DUBEDAT (Chicago)

" SLE Partition Functions"

Abstract: A prime motivation for the study of Schramm-Loewner Evolutions is their relation with scaling limits of critical discrete statistical mechanics models. Major information for these models is encoded in their partition functions. We discuss a continuous analogue of this, in particular in relation with SLE path properties and Gaussian formalism.

. Friday March 28: DOUBLE-HEADER

10:00 – 11:00 Prof. **Eckhard PLATEN** (Sydney)

The Law of the Minimal Price

Abstract: The talk introduces a general market setting under which the Law of One Price does no longer hold. Instead the Law of the Minimal Price will be derived, which for a range of contingent claims provides lower prices than suggested under the currently prevailing approach. This new law only requires the existence of the numeraire portfolio, which turns out to be the portfolio that maximizes expected logarithmic utility. In several ways the numeraire portfolio cannot be outperformed by any nonnegative portfolio. The new Law of the Minimal Price leads directly to the real world pricing formula, which uses the numeraire portfolio as numeraire and the real world probability measure as pricing measure when computing conditional expectations. The pricing and hedging of extreme maturity bonds illustrates that the price of a zero coupon bond, when obtained under the Law of the Minimal Price, can be far less expensive than when calculated under the risk neutral approach.

11:00 – 12:00 Prof. **Peter FRIZ** (Cambridge University)

Malliavin calculus and Rough Paths: Applications to Hoermander theory and stochastic PDEs

Abstract: Malliavin Calculus is about Sobolev-type regularity of functionals on Wiener space, the main example being the Ito map obtained by solving stochastic differential equations. Rough path

analysis is about strong regularity of solution to (possibly stochastic) differential equations. Combining arguments of both theories we discuss existence of a density for solutions to stochastic differential equations driven by a general class of non-degenerate Gaussian processes under Hoermander's condition. (*Joint work with T. Cass.)*

It turns out that these techniques also apply to a class of (stochastic) partial differential equations (with rough Gaussian noise); we take the opportunity to present some first result on what may turn into a theory of rough partial differential equations. *(Joint work with M. Caruana.)*

. **Friday April 4:** Prof. **Henrik HULT** (Brown)

Small but heavy-tailed random perturbations of deterministic and stochastic systems

Abstract: I will give an introduction to large deviations for stochastic processes with regularly varying tails. Then we shall focus on situations where a system is perturbed by a small but heavy-tailed (regularly varying) noise. First an ordinary differential equation is perturbed by a regularly varying noise ϵY and a functional large deviation result is obtained, as $\epsilon \to 0$. Then we shall consider small perturbations of a stochastic integral equation, and a large deviation result for hitting probabilities will be obtained. Finally, this result will be applied to an insurance problem to obtain the asymptotic decay of finite time ruin probabilities.

SPECIAL PROBABILITY SEMINAR

Monday, April 7th, 9:15-10:30 in Rm. 903 SSW

Professor Marc YOR, University of Paris & French Academy of Sciences

"The Riemann-Zeta Function and Random Matrix Theory"

Notes for the lecture can be found at

http://www.math.columbia.edu/~ik/riemann-zeta-yor08.pdf

. Friday April 11: Prof. **Edwin PERKINS** (UBC)

Degenerate Stochastic Differential Equations arising from Catalytic Branching Networks

We establish existence and uniqueness for the martingale problem associated with a system of degenerate finite dimensional SDE's representing a catalytic branching network. A special case of these results is required in recent work of Dawson, Greven, Den Hollander, Sun and Swart on mean field limits of block averages for 2-type branching models on a hierarchical group. The proofs make use of some new methods, including Cotlar's lemma to establish asymptotic orthogonality of the derivatives of an associated semigroup at different times, and a refined integration by parts technique for branching models.

MINERVA FOUNDATION LECTURES, SPRING 2008

Professor **Michel EMERY** (University of Strasbourg)

"Five Lectures on Manifold-Valued Semimartingales"

Tentative Times and Locations

Conference Room 1025 SSW, 1:00 – 2:30   Thursday April 10th, 17th

Conference Room 1025 SSW, 9:00 – 10:30   Thursday April 24th

Conference Room 1025 SSW, 1:00 – 2:30   Tuesday April 15th, 22nd

Prerequisites: Some familiarity with the basic definitions of continuous stochastic calculus will be assumed: previsible process, continuous martingale, continuous local martingale, continuous semimartingale, stochastic integration. No knowledge of differential geometry will be needed: the basic notions will be recalled, and connections, the only geometric tool that will be used, will appear only at the end.

Lecture 1: The notion of a *formal semimartingale* will be introduced and explained. The space of formal semimartingales contains the space of semimartingales; if S is a semimartingale and H a previsible process, the stochastic integral $\int H \, dS$ can always be defined as a formal semimartingale. This possibility of performing stochastic calculus with no integrability constraints on H gives much freedom, just as distributions liberate ordinary calculus from differentiability constraints. (By the way, the inventor of formal semimartingales is the same Laurent Schwartz whose name is associated to distributions.) Interesting fallouts of this notion: three definitions are made much simpler and more tractable, that of the space $L(S)$ of all previsible processes integrable with respect to a given semimartingale S, that of stochastic integration of vector-valued processes, and that of a σ-martingale.

We shall also recall some basic notions from (ordinary) differential geometry: differentiable manifolds, tangent and cotangent spaces and bundles.

Lecture 2: *Second order differential geometry* will be presented. It was devised by L.Schwartz to describe intrinsically the infinitesimal behavior of manifold-valued semimartingales, but can be understood in a non-probabilistic framework. The tangent space to a manifold at a point x consists of the speeds of all possible curves passing at x; similarly, the second-order counter-part to the tangent space contains the accelerations of all those curves. Second-order tangent and cotangent spaces and bundles will be introduced, as well as Schwartz morphisms (the natural morphisms between second-order tangent spaces). Purely geometric as they are, these objects will not be found in any textbook on differential geometry.

Then *manifold-valued semimartingales* are introduced, at last! Schwartz' principle says that if X is such a process, then dX (which, as everyone knows, is a convenient notation but does not exist) belongs to the realm of second-order geometry.

Lecture 3: Schwartz' principle will be illustrated by second-order stochastic integration, that is, integration of second-order covectors against a (manifold-valued) semimartingale. We shall then turn to a general (hence useful) theory of intrinsic second-order stochastic differential equations in manifolds. This concept will be made clearer by analogy with ordinary differential equations between manifolds: given two manifolds M and N, an ODE is a machine that transforms M-valued curves into N-valued ones, and similarly a SDE transforms M-valued semimartingales into N-valued ones.

By far, the most important examples and applications of this theory belong to *Stratonovich intrinsic stochastic calculus*, which was already known and practiced in the sixties, long before Schwartz pondered over second-order calculus around 1980. We shall see how purely geometric operations can transform ordinary 1-forms or ODEs into second-order forms or SDEs, giving rise to the ubiquitous Stratonovich transfer principle: geometric constructions on curves extend canonically to semimartingales.

Lecture 4: We shall elaborate on Stratonovich calculus considered as a particular case of Schwartz' calculus, and derive the theory of Stratonovich SDEs from the theory of second-order SDEs as presented in the previous lecture. We shall also describe a general approximation scheme by time-discretization, which yields Stratonovich objects.

A very important example consists of stochastic lifts and stochastic transport of vectors or tensors, of fundamental importance in practice. This will need the definition of a *connection*, a most fundamental tool in differential geometry.

Lecture 5: Much less well known than Stratonovich calculus, an *Ito intrinsic stochastic calculus* is also possible in manifolds. It requires some additional geometric structure, namely, a connection, but needs less smoothness than Stratonovich calculus. We shall introduce Ito stochastic integrals, Ito stochastic differential equations, the \Ito transfer principle, and an approximation scheme of Ito objects by time-discretization.

A few words will be said about manifold-valued martingales.

Lecture notes are available at **http://hal.archives-ouvertes.fr/hal-00145073/fr/**

SPECIAL MATHEMATICAL FINANCE PRACTITIONERS' SEMINAR

Thursday, April 17th, Room 207 Math 7:40-9:00

Dr. E. Robert FERNHOLZ (INTECH)

Modeling Equity Market Behavior

Abstract stock markets are stochastic models that exhibit some of the properties of real stock markets. In these markets, the stock capitalizations are modeled by Brownian motions with drift and variance processes that depend on the market weights or the ranks of the stocks. We assume that the stocks pay no dividends, and that there are no splits or mergers. We study a number of market properties, including long-term stability, the existence of arbitrage, and the distribution of capital.

The simplest market models, those that appear in classical finance, have constant drift and variance parameters. While these models can be useful for studying short-term phenomena, they are unstable over the long term, and, hence, unsuitable for long-term analysis. In fact, in a market of stocks with constant drift and variance parameters, after the passage of time virtually all the market capital will become concentrated into single stocks. For long-term stability, variable drift and variance processes are needed, and we consider abstract markets that are stabilized either by volatility or by the use of parameters that are based on rank.

Volatility-stabilized markets are abstract markets in which the variance of the stocks is greater for smaller stocks. This property holds for real stock markets, and we show that it results in a form of long-term stability. In some of these markets, the greater volatility of the smaller stocks can be exploited by portfolios that systematically overweight these stocks, and this creates an opportunity for arbitrage.

Markets of stocks with drift and variance parameters that depend on rank can also be stable over the long term. Roughly speaking, if the lower-ranked stocks drift upward faster than the larger stocks, then the market will be stable over the long term. We can create markets of this type that have stable capital distributions similar to the capital distributions of real stock markets, however, the dynamic behavior of these markets is likely to be quite complicated, and there are many open questions regarding them.

. Friday April 18: DOUBLE-HEADER

10:00 – 11:00 Prof. **Rama CONT** (Columbia)

Recovering default intensity from CDO markets by entropy minimization and intensity control

Abstract: Pricing models for portfolio credit derivatives such as CDOs involves the construction of a marked point process for the losses due to defaults which is compatible with a set of observations of market spreads for CDO tranches. We propose a stable nonparametric algorithm to solve this inverse problem by mapping it into a stochastic control problem. We formalize the problem in terms of minimization of relative entropy with respect to a prior jump process under calibration constraints and use convex duality techniques to solve the problem. The dual problem is shown to be an intensity control problem. We show that the corresponding nonlinear Hamilton Jacobi system of differential equations can be represented in terms of a nonlinear transform of a linear system of ODEs and thus easily solved. Our method allows to construct a Markovian jump process for defaults which leads to CDO tranche spreads consistent with the observations. We illustrate our method using ITRAXX index data: our results reveal strong evidence for the dependence of loss transitions rates on the past number of defaults, thus offering quantitative evidence for contagion effects in the risk-neutral loss process.

11:00 – 12:00 Prof. **Michel EMERY** (Strasbourg)

On Maximal Brownian Motions

Abstract: Many difficult questions stand open in the theory of filtered probability spaces. For instance, call $\BB^\infty$ a filtration generated by countably many independent BM's, and consider a sub-filtration $\FF$ of $\BB^\infty$. Suppose that (i) every $\FF$-martingale is a $\BB^\infty$-martingale, and (ii) $\FF$ has the previsible representation property with respect to some $\FF$-BM. *Is $\FF$ necessarily generated by some BM?*

The talk will describe maximal one-dimensional $\BB^2$-BMs, where $\BB^2$ is now the filtration created by a fixed two-dimensional BM. (Maximality is understood in the following sense: no other one-dimensional $\BB^2$-BM has a strictly larger filtration.)
Some necessary or sufficient conditions for maximality will be given.

We consider this a first step toward the question mentioned above, and other similar ones. Already in this simpler framework, many tantalizing problems are unsolved. For instance, if a one-dimensional $\BB^2$-BM X is maximal, does there necessarily exist another, independent $\BB^2$-BM Y such that the two-dimensional BM (X,Y) generates the whole filtration $\BB^2$? *(Joint work with J. Brossard and C. Leuridan).*

SPECIAL PROBABILITY SEMINAR

Wednesday April 23rd, 5:00-6:00
Room 520 Mathematics

Tea at 4:40 in the Math Bldg Lounge

Professor Alexei BORODIN (Caltech)

"Random surfaces in dimensions two, three, and four"

Abstract: The goal of the talk is to survey recent results on various classes of random surfaces with focus on their asymptotic behavior. Examples include random partitions, driven interacting particle systems, and random tilings. No preliminary knowledge of the material will be assumed.

. Friday May 2nd: Dr. **Jose BLANCHET** (Columbia)

Efficient Simulation for Random Walks Avoiding Hard Obstacles

Abstract: In this talk we will describe an asymptotically optimal (in a precise sense) importance sampling estimator for the probability that, for a long time, a random walk avoids obstacles randomly placed in the space. Classical large deviations results by Donsker and Varadhan provide logarithmic asymptotics for such probabilities. However, the large deviations techniques typically applied to this problem are indirect in the sense that one avoids a direct construction of "the most likely path" based on an appropriate change-of-measure -- a construction that often provides the key elements for the optimal design of importance sampling algorithms. It turns out that the optimal trajectory followed by the random walk to avoid the obstacles involves keeping track of the range visited by the random walk and therefore the state-space increases, which makes the design of an efficient algorithm challenging. The talk is about describing how can we construct algorithmically an importance sampling estimator that keeps track of the history of the process in a way that mimics the optimal path close enough to achieve asymptotically optimal variance properties. *(Joint work with Paul Dupuis.)*

. Friday May 9: 11:00 – 12:00 Prof. **Jerzy ZABCZYK** (Warsaw)

SPDE Formulation of Interest-Rate Models with Levy Noise

Abstract: The talk is concerned with a class of stochastic PDEs describing the time evolution of forward curves in the bond market. The noise is modeled by Levy processes and risk free and defaultable bonds are studied. Conditions for existence of solutions, positivity, and long-time behavior are presented. Special attention is paid to models with volatilities depending linearly on forward curves. In some cases closed form solutions are found. *(Joint work with A. Rusinek and S. Peszat.)*

Department of Statistics

Summer 2008

Mini Course: Topics in Probability

May 27, 28, 29 and June 10, 11, 12

Time: 1:00 – 2:30 pm

Instructor: Professor Jean BERTOIN, Universite Paris VI

Syllabus

A. An introduction to subordinators and Levy processes.

1. An overview of Levy processes
   (infinitely divisible laws, Levy-Khintchine formula, structure of the jumps, Markov property and applications, some path properties).

2. Some aspect of subordinators
   Levy-Khintchine-Ito's theory on R_+, examples, Bochner's subordination, renewal theory in continuous times).

3. Levy processes with no negative jumps
   (extrema of Levy processes with no negative jumps, connection with subordinators, formulas of fluctuation theory, connection with branching processes).

4. Exponential functionals (method of moments for the law of the exponential functional of a subordinator, case of general Levy processes, connection with self-similar Markov processes, Stieltjes' problem of moments).

5. Some aspects of Levy processes in mathematical finance and insurance
   (models with jumps, stochastic volatility, ruin problem,...).

B. On exchangeable coalescents

1. Various notions of partitions, Kingman's theory of exchangeable random partitions

2. Poisson-Dirichlet random partitions, Kingman's coalescent

3. Exchangeable coalescents: definition and first properties, Rates of coagulation and construction.

4. Characterization of coagulation rates, Masses in exchangeable coalescents

5. Simple coalescents and generalized Fleming-Viot processes, The Bolthausen-Sznitman coalescent .

C. Some aspects of self-similar fragmentations

Fragmentation phenomena can be observed in many sciences at a great variety of scales. To give just a few examples, let us simply mention the studies of stellar fragments and meteoroids in astrophysics, fractures and earthquakes in geophysics, breaking of crystals in crystallography, degradation of large polymer chains in chemistry, fission of atoms in nuclear physics, fragmentation of a hard drive in computer science, ..., not to mention crushing in the mining industry.

The purpose of this mini-course is to introduce a mathematical framework which might serve as model for situations in which fragmentations occurs randomly and repeatedly as time passes.

We shall first construct a large family of fragmentation processes in connection with branching Markov chains in continuous times. Then, we shall investigate fine properties of such processes depending on their characteristics. In particular, we shall point at the Phenomenon of formation of dust when the index of self-similarity is negative, and to a homogeneization type property for positive indices.

D. The rate of growth of Levy processes

This mini-course will be based on a recent paper jointly with Ron DONEY (Manchester) and Ross MALLER (Canberra) about the local rate of growth of general Levy processes. It provides a refinement of old results due to Blumenthal and Getoor who associate to every Levy process an "upper-index" β and related the rate of growth of the Levy process to that of a stable process with index β.

References:

Bertoin, J., Doney, R. and Maller, R. (2007) Passage of L´evy processes across power law boundaries at small times. To appear in Annals of Probability. http://www.imstat.org/aop/futurepapers.htm

Blumenthal, R.M. and Getoor, R.K. (1961) Sample functions of stochastic processes with stationary independent increments. J. Math. Mech.10, 492-516.