

Spring Semester 2006

Tue-Thu 10:35 – 11:50 in 1025 SSW, 1255 Amsterdam Ave.

Professor Ioannis Karatzas

G8243: TOPICS IN ADVANCED PROBABILITY

Suggested topics include:

. Notion and properties of *Local Time*, for Brownian Motion and for Semimartingales. The formulae of D. Williams; the theorems of Ray & Knight. Elements of Excursion Theory.

Study of *Stochastic Differential Equations*: Weak and strong solutions, pathwise uniqueness and uniqueness in law, the Yamada-Watanabe theory. Comparison and approximation theorems. Martingale problems, the theory of Stroock & Varadhan; strong Markov property. The support theorem for solutions of SDE's. Time-reversal of diffusions.

. One-dimensional diffusions: scale function and speed measure, the Engelbert-Schmidt theory, the Feller and Khas'minskii tests for explosion, detailed study of the Bessel SDE. Properties of Bessel processes: the Pitman theorem, the D.Williams path decomposition.

. Special topics: Zvonkin and Tsirel'son examples, the Doss-Sussmann pathwise construction. Pure and conformal martingales. The "most probable path" of Brownian motion with drift.

. Martingales depending on a parameter; the Ito-Wenzell formula. Stochastic Flows of Diffeomorphisms.

. Backwards Stochastic Differential Equations: Basic notions and properties, connections with partial differential equations.

Introduction to the *General Theory of Processes*: Optionality, Predictability, Filtering.

Introduction to the *Malliavin Calculus*: multiple Wiener integrals, the Skorohod integral and the Malliavin derivative, the Clark-Ocone formula, Hormander's hypoellipticity condition. The Ornstein-Uhlenbeck semigroup; hypercontractivity.

. Introduction to *Stochastic Control* and to *Optimal Stopping*. Probabilistic and analytical approaches.

. Additional topics will reflect the interests of the audience.

RECOMMENDED TEXTS AND MONOGRAPHS:

N. IKEDA & S. WATANABE (1981) *Stochastic Differential Equations and Diffusion Processes*. North Holland, Amsterdam & Kodansha Ltd, Tokyo.

I. KARATZAS & S.E. SHREVE (1987) *Brownian Motion and Stochastic Calculus*. Springer-Verlag, New York.

H. KUNITA (1990) *Stochastic Flows and Stochastic Differential Equations*. Cambridge University Press.

D. NUALART (1995) *The Malliavin Calculus and Related Topics*. Springer, New York.

D. REVUZ & M. YOR (1991) *Continuous Martingales and Brownian Motion*. Springer-Verlag, New York.

L.C.G. ROGERS & D. WILLIAMS (1987) *Diffusions, Markov Processes and Martingales, Vol. II: Ito Calculus*. J. Wiley & Sons, Chichester and New York.