1. Show that each of the following are linear operators on $\mathbb{R}^2$. Describe geometrically what each transformation accomplishes, and find the matrix associated with this transformation.

   (a) $L(x) = (-x_2, x_1)$
   (b) $L(x) = x_1 e_1$
   (c) $L(x) = \frac{3}{2}x$

2. Let $L : \mathbb{R}^3 \mapsto \mathbb{R}^3$ be a linear operator such that

   $L((1, 1, 1)^T) = (-2, 3, 0)^T$, $L((-1, 1, 0)^T) = (0, 0, 0)^T$, $L((1, 0, 1)^T) = (-1, 0, 0)^T$

   Determine the value of $L((-2, 4, 1)^T)$.

3. Let $L$ be the linear operator on $\mathbb{R}^2$ defined by

   $L(x) = (x_1 \cos \alpha - x_2 \sin \alpha, x_1 \sin \alpha + x_2 \cos \alpha)^T$

   Express $x_1$, $x_2$ and $L(x)$ in terms of polar coordinates. Describe geometrically the effect of the linear transformation.

4. Let $a$ be a fixed nonzero vector in $\mathbb{R}^2$. A mapping of the form

   $L(x) = x + a$

   is called a translation. Show that a translation is not a linear operator. Illustrate geometrically the effect of the translation.

5. Let $C$ be a fixed $n \times n$ matrix. Determine whether the following are linear operators on $\mathbb{R}^n \times \mathbb{R}^n$.

   (a) $L(A) = A^T$
   (b) $L(A) = CA + AC$
   (c) $L(A) = A + C$

6. If $L$ is a linear transformation from $V$ to $W$. Use mathematical induction to show that

   $L(\alpha_1 v_1 + \cdots + \alpha_n v_n) = \alpha_1 L(v_1) + \cdots + \alpha_n L(v_n)$

7. Determine the kernel and range of each of the following linear operators on $\mathbb{P}^3$.

   (a) $L(p(x)) = xp'(x)$
   (b) $L(p(x)) = p(0) + p(1)x$
8. For each of the linear transformations in the question above, find the matrix representing this transformation with respect to the basis \( \{1, x, x^2\} \) and the basis \( \{1, 1 + x, 1 + x + x^2\} \).

9. Find the standard matrix representation for each of the following linear operators.

(a) \( L \) is the linear transformation that rotates a vector in \( \mathbb{R}^2 \) by 45° clockwise.

(b) \( L \) is the linear transformation that doubles the vector \( x \) in \( \mathbb{R}^3 \) and then reflects it across the \( x - y \) plane.

(c) \( L \) is the linear transformation that rotates a vector in \( \mathbb{R}^3 \) by 30° clockwise around the \( z \)-axis.

10. The trace of an \( n \times n \) matrix \( A \), denoted \( tr(A) \), is the sum of its diagonal entries; that is,

\[
tr(A) = a_{11} + a_{22} + \cdots + a_{nn}
\]

Show that

(a) \( tr(AB) = tr(BA) \)

(b) If \( A \) is similar to \( B \) then \( tr(A) = tr(B) \)