1. (2 points each) Let

\[ A = \{ \pi, \frac{1}{8}, \sqrt[3]{-27}, 12 \} \]
\[ B = \{ 0, 3, -3, 9, -9 \} \]
\[ C = \{ x | x^3 = -27 \} \]

Find the indicated set.

(a) \( A \cap \mathbb{R} \)
   Answer: \( A \)

(b) \( A \cap C \)
   Answer: \( \{-3\} \)

(c) \( B \cap C \)
   Answer: \( \{-3\} \)

(d) \( A \cup B \)
   Answer: \( \{ \pi, \frac{1}{8}, -3, 12, 0, 3, 9, -9 \} \)

2. (2 points each) Evaluate each number.

(a) \( \left( \frac{2}{7} \right)^0 \left( \frac{4}{49} \right)^{-\frac{1}{2}} \)
   Answer: \( \frac{7}{2} \)
(b) \((\frac{1}{4})^{-2}\)
Answer: 16

(c) \((\sqrt{23})(\sqrt{23})(23^{1/3})(23^{2/3})\)
Answer: 529

(d) \(1 + |2 - | - 3||\)
Answer: 2
3. (3 points each) Simplify the expression and eliminate any negative exponents. Assume that all letters denote positive numbers.

(a) \( \left( \frac{x^4 y}{y^2 z^2} \right)^{5/3} \)
Answer: \( \frac{x^6}{y^{10}} \sqrt[3]{z^{10}} \)

(b) \( (a^3 b^7)^{-2} \)
Answer: \( \frac{1}{a^6 b^{14}} \)
4. (4 points each) Perform the indicated operations and simplify.

(a) \((x^4 y^2 - y^3)(x^2 + \sqrt{y})(x^2 - \sqrt{y})\)
   Answer: \(x^8 y^2 - 2x^4 y^3 + y^4\)

(b) \((x^2 - a)(x^2 + a^2)\)
   Answer: \(x^4 - ax^2 + a^2 x^2 - a^3\)

5. (4 points each) Factor the expression completely

(a) \(x^6 + 2x^5 + x^4\)
   Answer: \(x^4(x + 1)(x + 1)\)
(b) \(4x^4z^2 - 25y^2z^2\)
Answer: \(z^2(2x^2 - 5y)(2x^2 + 5y)\)

(c) \(27x^3 - 8\)
Answer: \((3x - 2)(9x^2 + 6x + 4)\)
6. (4 points each) Simplify the expression

(a) \[ \frac{y^2 + 8y + 16}{y + 4} \div \frac{y^2 - 16}{y - 4} \]
Answer: 1

(b) \[ \frac{a - \frac{4}{a+2}}{2 - a} \]
Answer: \[ \frac{a^2 + 2a - 4}{(a+2)(2-a)} \]
7. (4 points each) Find all real solutions to the following equations.

(a) \( \frac{z}{2} + 3 = \frac{3}{4}z + 7 \)
Answer: \( z = -16 \)

(b) \( 3x + 2 = -7x + 1 \)
Answer: \( x = -\frac{1}{10} \)

(c) \( x^2 - x - 5 = 0 \)
Answer: \( x = \frac{1 + \sqrt{21}}{2}, \frac{1 - \sqrt{21}}{2} \)
(d) $|x| + 3 = 4$ Answer: $x = 1, -1$

(e) $x^2 - x = 6$
Answer: $x = 3, -2$

(f) \[
\frac{x}{x - 1} - \frac{2}{x + 1} = \frac{4}{x^2 - 1}
\]
Answer: $x = 2$
(g) \( \sqrt[3]{2x^3 - 1} + 2 = x^2 + 2 \) Answer: \( x = 1 \)
8. (4 points each) Solve the following inequalities. Express the solution in interval notation.

(a) \( x^2 + 5x > -6 \)
   \[ \text{Answer: } (-\infty, -3) \cup (-2, \infty) \]

(b) \( x \leq \frac{9}{2} \)
   \[ \text{Answer: } (-\infty, -3] \cup (0, 3] \]

(c) \( |x| + 3 < 4 \)
   \[ \text{Answer: } (-1, 1) \]
9. (4 points) Let \( A = (1, 1) \) and \( B = (2, -1) \). Find the equation of a line through \( A \) and \( B \), and graph the line.

Answer: The equation of the line is \( y = -2x + 3 \) (graph omitted)
10. (6 points) Sketch the graph of the function

\[ g(x) = \begin{cases} 
4x - 5 & \text{if } x < 0 \\
(x - 2)^2 & \text{if } x \geq 0 
\end{cases} \]

Answer: Graph should look like a line for \( x < 0 \) and a parabola for \( x \geq 0 \). At \( x = 0 \) the function is 4, so the point \((0, 4)\) should be a solid dot, while the point \((0, -5)\) should be an open circle. The graph jumps at \( x = 0 \).
11. (3 points each) Let
\[ f(x) = \begin{cases} 
\sqrt{x} & \text{if } x \geq 9 \\
x^2 - 3 & \text{if } x < 0 \\
3x & \text{if } 1 < x < 9
\end{cases} \]

What are the following values of \( f \)?

(a) \( f(9) \) Answer: 3

(b) \( f(-1) \) Answer: -2

(c) \( f(4) \) Answer: 12

(d) \( f(a^2) \) where \( a > 3 \) Answer: \( a \)

12. (4 points) What is a function?
Answer: A function is a rule that assigns to each element \( x \) of the domain a unique element \( y \) of the range.

13. (4 points) What does it mean for a function \( f \) to be one-to-one?
Answer: A function is one-to-one if no two input values give the same output. In other words, for all \( x_1 \neq x_2 \) in the domain we also have \( f(x_1) \neq f(x_2) \).
14. (4 points each) Suppose that \( f = \sqrt{x} \)

Sketch a graph of each of the following functions. Be sure to label the axes.

(a) \( f(x - 1) \)

Answer: shift to the right one unit

(b) \( f(x) - 1 \)

Answer: shift down one unit
15. (3 points each) Suppose that \( f, g \) and \( h \) are one-to-one functions and that
\[
\begin{align*}
  f(3) &= 4 & g(1) &= 2 & h(2) &= 3 \\
  f(4) &= 2 & g(3) &= 6 & h(4) &= 1 \\
  f(6) &= 3 & g(-3) &= 6 & h(1) &= 8
\end{align*}
\]
Find the following values:

(a) \( f^{-1}(2) \) Answer: 4

(b) \( f \circ f(6) \) Answer: 4

(c) \( g^{-1} \circ f(4) \) Answer: 1

(d) \( h^{-1} \circ h(5) \) Answer: 5

(e) \( f \circ f \circ h(2) \) Answer: 2

(f) \( h^{-1} \circ g^{-1}(2) \) Answer: 4
16. (8 points) A rancher with 750 ft of fencing want to enclose a rectangular area and then divide it into four pens with fencing parallel to one side of the rectangle. (i) Find a function that models the total area of the four pens, (ii) Find the largest possible total area of the two pens.

Answer: (i) Two possibilities, write the area as a function of width $w$ as $A = \frac{750w - 5w^2}{2}$, or as a function of length $L$ as $A = \frac{750L - 2L}{5}$.

(ii) The maximum occurs for width $w = 75$, the maximum area is thus 14212.5 ft$^2$. 

17. (8 points) A factory is to be built on a lot measuring 180 ft by 240 ft. A local building code specifies that a lawn of uniform width and equal in area to the factory must surround the factory. What must the width of this lawn be, and what are the dimensions of the factory?

Answer: The dimensions of the factory are 120 ft by 180 ft and the width of the lawn is 30 ft.