

A Sometimes Funny Book Supposedly about Infinity

A Review of Everything and More

Reviewed by Michael Harris

Everything and More

David Foster Wallace

W. W. Norton & Company, 2003

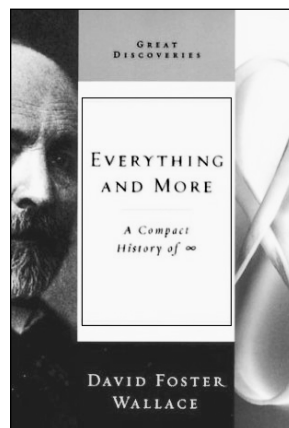
320 pages, \$23.95, ISBN 0-393-00338-8

Exalted perhaps in the wake of the World Trade Center attack and suspecting probably that the American population would be satisfied with no response that fit within the limits of human comprehension, “someone, somewhere” in the Pentagon—the quotation is from Donald Rumsfeld—proposed “Infinite Justice” in late 2001 to designate the imminent campaign in Afghanistan. It was soon pointed out that many people, not least among those for whom the campaign was intended as an object lesson, considered infinity a divine prerogative, and the title was scrapped in favor of “Enduring Freedom”. Instead of a guarantee both mathematical and eschatological, the world would have to settle for a pretty good insurance policy.

In the interim, more than one skeptic¹ had time to note the initial designation’s inadvertent echo of *Infinite Jest*. It says something about our author’s standing in American letters, at least among the wired, that the reference was not to Shakespeare’s Poor Yorick, but rather to the title of David Foster Wallace’s 1996 novel. On everybody’s Ten Best list, qualified as “world-historical” by Frederic

Michael Harris is professor of mathematics at Université Paris 7. His email address is harris@math.jussieu.fr.

I thank Boban Velicković for a critical reading of E&M’s sections on set theory, and Jordan Ellenberg for showing me an early manuscript of his review of E&M in Seed.



Jameson,² the 1,079-page *Infinite Jest* (IJ) propelled Wallace (DFW) immediately—and deservedly—to the front rank of American writers. Alongside his fiction, DFW has developed an intensely original and provocative style as essayist and often profound observer of contemporary U.S. culture. His best essays of the '90s are collected in *A Suppos-*

edly Fun Thing I'll Never Do Again (ASFTINDA), the very first page of which reports Wallace’s adolescent “jones for mathematics”. In between two collections of short stories, DFW has taken the time to write *Everything and More* (E&M), ostensibly about the events leading up to and the circumstances surrounding Georg Cantor’s creation of his theory of infinite sets.

Wallace, future classic, will have his literary biographers, who will probably have little patience with the professional mathematician who objects to E&M on the grounds that it not only lacks a clear sense of direction (try figuring out what happens in IJ without a guide) but is laced through and through with blunders of every magnitude. It does not help that the two most strongly negative reviews of E&M thus far³ were written by authors of what sympathetic biographers might construe as rival

books about infinity. The returning exile discovers that popular books about infinity are already available in his home market. Rather a lot of them, it turns out. Yet another one—Brian Clegg’s *Infinity: The Quest to Think the Unthinkable*—appeared almost simultaneously with E&M, and the two books were reviewed together in the *Guardian* by the eminent critic Frank Kermode.

Though there seems to be no limit to the demand for such books, a good chunk of the table of contents seems predetermined, a fact which may help you write your own book on infinity and which may in itself constitute an a priori proof of the existence of infinity.⁴ Nevertheless, each new entry in the field needs to establish its distinctive brand identity. E&M is the first in a series called Great Discoveries, a joint venture between W. W. Norton and New York literato James Atlas. Many sorts of greatness sit comfortably on Atlas’s shoulders: he piloted the series Great Lives, devoted to Alexander-level greatness, at Viking Penguin, and now his new outlet, Atlas Books,⁵ has plans for a joint partnership called Eminent Lives with HarperCollins⁶ (featuring Alexander himself); another partnership with W. W. Norton (and Profile Books in the UK) entitled Enterprise: Great Innovations in Business;⁷ and Great Discoveries, dedicated to science.

The common feature of all these celebrations of greatness is the pairing of “today’s top writers”⁸ with subjects indisputably great. “David Foster Wallace, for example, has a real following, so this book will be embraced by his fans,” according to Ed Barber, senior editor at Norton.⁹ Barber guessed right: when I checked about one month after the book’s release, E&M was listed 190th among Amazon’s bestsellers. In early November it was prominently displayed in at least two East Coast airport bookshops. Three months later it is still in the airports, is in the respectable 3,000s on Amazon, and graces front tables in bookstores around Harvard Square and beyond.

With answers like this, questions are clearly beside the point. One should not dismiss E&M on the grounds of mercenary considerations alone, however. In the universe of Wallace’s IJ it is not merely public-domain concepts like Great Discoveries but time itself, or at least the calendar, that has been branded, and one should assume this author is keenly sensitive to the dangers of being turned into a brand name himself. And the very existence of a series like Great Discoveries poses a question that is particularly acute for mathematicians: Is it possible to write a book about our subject that is not only comprehensible and accurate in the essentials but also has literary merit? Can “the narrative approach to science”¹⁰ work for mathematics? Occasionally one comes across a readable and reliable mathematical popularization that is also well written. But for an experimental writer like

DFW, the stakes, and therefore the hopes, are even greater: creation of a new literary genre within what Wallace calls “pop technical writing”, in which the sense and depth of mathematics can be expressed creatively and readably with minimal loss.

One problem is that mathematical truth is rarely graceful when transcribed into natural language. Wallace makes this point explicitly in several passages in E&M: “...some technical words have highly specific denotations that no synonym can capture. Which means that, especially respecting certain high-tech proper nouns, abbreviation is the only way to achieve any kind of variation at all” (p. 3). Not every reader will find Wallace’s abbreviations graceful, in or out of context, but the use of abbreviations and invented acronyms (often left to the reader to figure out in his other books; here he provides a glossary)¹¹ is an immediately recognizable feature of his style and contributes to its characteristic fusion of high and low cultures and of good and bad taste. This sort of ambivalent irreverence for high cultural traditions is familiar from alternative newspapers like the *Village Voice* and *LA Weekly* and the ambiguous status of rock critics and comic strip artists, for example; DFW himself has associated it with TV. The jury is out on whether this makes DFW a “postmodern” author, but it does make him a natural candidate for inventing a pop mathematics writing that would be to university mathematics as rock criticism is to symphonic music, and it is why I volunteered for this assignment. Among other qualities distinguishing Wallace from most of his post- (or post-post-?) modern contemporaries are his breadth of knowledge and interest, frequently described as “encyclopedic”, and a brilliant talent for pastiche, displayed to great effect in IJ. He also has his share of idiosyncracies: his combination of a professorial tone¹² that brooks no contradiction¹³ with deflationary words like “basically”, “pretty much”, “sort of”, “stuff”, or “shit”¹⁴ that create the illusion of a conversation; a prodigious use of run-on sentences; the aforementioned acronyms; and especially the profusion of footnotes that supplement and in some cases contradict the main narrative.¹⁵

DFW places his book in its contemporary pop cultural context when he announces what kind of book about infinity it is not:

The Mentally Ill Mathematician seems now in some ways to be what the Knight Errant, Mortified Saint, Tortured Artist, and Mad Scientist have been for other eras: sort of our Prometheus, the one who goes to forbidden places and returns with gifts we can all use but he alone pays for. (E&M, pp. 6–7)

He will instead concentrate on “Cantor’s work and its context...so totally interesting and beautiful that there’s no need for breathless Prometheusizing of the poor guy’s life” (p. 7). “The aim is to discuss these achievements in such a way that they’re vivid and comprehensible to readers who do not have pro grade backgrounds and expertise” (p. 1).

So that is the promise: “probably the most important novelist of his generation”¹⁶ will engage the actual mathematical content of Cantor’s theory of infinity while not ceasing to write in an avant-garde way. And instead of doing that, he begins with a long discussion of metaphysics and abstraction, on the grounds that infinity is “sort of the ultimate in drawing away from actual experience.” Unlike many authors of recent popularizations, DFW makes a point of failing to develop mathematicians’ characters, letting their work speak for itself. But in order to embed abstraction in lived experience, he is forced briefly to liberate the narrator from his primary role as professorial Virgil leading the reader’s Dante to Cantor’s paradise (from which Hilbert promised we would never be expelled) and to allow the narrator—i.e., himself—to adopt the posture of a human being beset by abstract thinking who suddenly wonders whether he is justified in believing that the floor will support him when he gets out of bed in the morning. To illustrate the pitfalls of induction from experience and belief in the reality of abstract ideas, he introduces the book’s only other fully human character:

There were four chickens in a wire coop off the garage, the brightest of whom was called Mr. Chicken. Every morning the farm’s hired man’s appearance in the coop area with a certain burlap sack caused Mr. Chicken to get excited and start doing warmup-pecks at the ground, because he knew it was feeding time. It was always around the same time t every morning, and Mr. Chicken had figured out that t (man + sack) = food, and thus was confidently doing his warmup-pecks on that last Sunday morning when the hired man suddenly reached out and grabbed Mr. Chicken and in one smooth motion wrung his neck and put him in the burlap sack and bore him off to the kitchen...[However,] Mr. Chicken appears...actually to have been correct—according to the Principle of Induction—in expecting nothing but breakfast from that $(n + 1)$ th appearance of man + sack at t . Something about the fact that Mr. Chicken not only didn’t suspect a thing but appears to have been wholly justified in not suspecting a thing—this seems concretely creepy and upsetting.

I have quoted this passage at length as a representative sample of DFW’s prose and of his informal approach to mathematical formalism, and to illustrate that his preoccupation with infinity here, and presumably in his fiction as well, is ultimately metaphysical. As it should be. Wallace distinguishes between what you “know” and what you really know. Mr. Chicken’s predicament, and ours as finite beings, lies in the difference between the two. The metaphysical problem of infinity, and the reason for its historically controversial status in mathematics, is whether and by whom or what infinity can be really known and whether it really matters.¹⁷ This problem has been central to Western philosophy throughout its history. Thus Augustine waxed indignant at the suggestion “that things infinite are past God’s knowledge,” Aquinas attempted to prove that “The existence of an actually infinite multitude is impossible,” and Hume that “The capacity of the mind is not infinite.”¹⁸ In his most extensive philosophical justification of transfinite numbers, Wallace recalls, Cantor actually quoted Aquinas’s arguments in detail and was particularly pleased to regard his own work as a refutation of these, “the weightiest” arguments ever advanced against infinity.¹⁹ In one of his best passages, Wallace regrets, eloquently, that the controversy has been whitewashed in the standard math curriculum. “Rarely do math classes ever tell us whether a certain formula is truly significant, or why, or where it came from, or what was at stake....That we end up not even knowing that we don’t know is the really insidious part of most math classes” (p. 52 and note 9).

The most carefully argued²⁰ section of E&M recapitulates the origins of the metaphysical problem of infinity in Zeno’s paradoxes—“immeasurably subtle and profound,” according to Bertrand Russell (quoted on p. 48), “nutcrunching” according to DFW—and their attempted resolution by Aristotle. Wallace asserts, I think correctly, that dismissing Zeno’s dichotomy paradox by invoking the standard arguments for the summation of a convergent geometric series, in this case $\sum(1/2)^n = 1$, is “complex, formally sexy,²¹ technically correct, and deeply trivial. Along the lines of ‘Because it’s illegal’ as an answer to ‘Why is it wrong to kill?’” (pp. 51–52).²² His objections to Aristotle are more acerbic. Aristotle’s response to Zeno’s arguments, briefly, consisted in denying infinity anything more than a potential, as opposed to actual, existence. In DFW’s reading, this was the “most efficient cause” behind the circumstance that “Europe had to wait nineteen centuries for the actual calculus, differential geometry, and analysis” (pp. 86–87). A few more centuries passed before Cantor finally placed actual infinite sets on a rigorous footing.

A two-sentence summary of the theme of E&M might be: Aristotle, for DFW, was content to “know” about infinity; after Cantor it became possible to know. Or did it?

“Real appreciation” of Cantor’s innovations “requires understanding the concepts and problems that gave rise to set theory and made transfinite math significant in Hardy’s sense” of “being connected, in a natural and illuminating way, with a large complex of other mathematical ideas.”²³ And so, accompanied by the wraithlike honors high school math teacher “Dr. Goris”, who probably really exists (and presumably presided over Wallace’s “jones for mathematics”), though he serves several obvious structural functions in the narrative, DFW embarks with jolly ferocity on a 200-page historical and technical discussion of mathematical analysis.²⁴

Rudy Rucker’s judgment that “Nothing makes sense”²⁵ from this point on is surely too harsh, but this is where Wallace begins to get into serious trouble. Most mathematicians are aware that Cantor was led to develop set theory in the course of his work on pointwise convergence of trigonometric series. Cantor’s Uniqueness Theorem asserted that two trigonometric series that converge pointwise everywhere on the unit interval to the same function have the same coefficients. His subsequent generalizations of this theorem replaced the condition “everywhere” by the complements of certain kinds of subsets of the real numbers, and in order to state his theorems he therefore requires theories of the real numbers and the subsets thereof. Though pointwise convergence of trigonometric series is a topic of little obvious metaphysical interest and is no longer as central a preoccupation as it was in Cantor’s day,²⁶ Wallace spends nearly half his book preparing for, stating, and concluding (on pp. 233–234) with an irrelevant argument purporting to be a proof of the most general form of Cantor’s Uniqueness Theorem.²⁷

The *New York Times* reviewer may have had these sections in mind when he admitted, in the course of a generally positive review, that it contains “a smattering of technical infelicities.”²⁸ A mathematician reading E&M is unlikely to be so forgiving of DFW’s blunders.²⁹ Some examples: The Weierstrass approach to limits is illustrated by a solution to Zeno’s dichotomy paradox in which $\delta = 1$ for any ϵ (p. 190 ff). The Extreme Value Theorem is used to prove, Zeno be damned, that on any time interval $[t_1, t_2]$ the “time function” [*sic*] has an absolute minimum t_m which is “mathematically speaking, the very next instant after t_1 ” (p. 190). Although the continuum hypothesis is a principal theme and although it is correctly stated several times, DFW also claims three times that the continuum hypothesis is the assertion that the continuum has the cardinality of the power set of

the natural numbers. A spurious distinction is drawn between “point-set theory” and “abstract set theory”; a presentation of Zermelo-Fraenkel set theory somehow fuses the Axiom Schema of Comprehension (which he calls the Limited Abstraction Principle) with the Axiom of Infinity; Kronecker is said to be most famous for his Delta Function (“which in some ways anticipates the binary math of modern digitation” [*sic*]); Dedekind is identified as a prototypical Platonist but also quoted to the effect that “Numbers are free creations of the human mind;” Cantor and Gödel are asserted to be respectively the most important mathematicians of the nineteenth and twentieth centuries (p. 5).

Journalist Jim Holt, writing in the *New Yorker*, pointed out a few of these infelicities and offered the curious hypothesis that Wallace is pulling our legs—E&M is actually “a sly send-up of pop technical writing.” I would not put it past the author. Much of postmodern literary theory is concerned with a topic Wallace considers in ASFTINDA, that of undermining the authority of the author or of the narrative voice, and there is hardly a better way to do that than to make the author manifestly unreliable. DFW himself warns, without providing details, that there have been “infelicities permitted” (note 2, p. 126) in order to make the arguments accessible to readers with little background in mathematics. It may be no accident that the peculiar ranking of Cantor and Gödel mentioned above is followed immediately by the assertion that Hilbert was “the world’s #1 mathematician” in the year 1900....

A characteristic feature of DFW’s style in E&M is to disrupt narrative continuity (of which there is very little in IJ) with technical or historical digressions that he calls “interpolations”. Here’s an illustration. A college student and fellow passenger on the Boston subway spotted me reading E&M and asked me what I thought of the book. My response was substantially equivalent to the present review. She had also read the book, of which she understood very little, but “I assumed it was me.” Reading IJ had also been hard work, so she had no reason to expect E&M to reveal its secrets without effort. True enough. Much that appears to be casual in IJ turns out to be crucial to understanding the structure as well as the author’s intentions.³⁰ The same may conceivably be true of E&M, that the book only appears to be about the mathematics of infinity but that the author’s real purpose is elsewhere: to tell a story or a joke whose point this reviewer has not worked hard enough to grasp. I have worried about this sometimes. But it has not kept me from getting out of bed in the morning.

One problem with Holt’s interpretation, appealing as it is, is that what might be taken as DFW’s artistic manifesto³¹ comes out explicitly in favor of sincerity over hip irony and cynicism. With

regard to mathematics, Wallace's attitude toward Dr. Goris is never short of reverential, and footnote 324 of IJ echoes the apparently sincere preface to E&M when the (admittedly ambiguous) character Pemulis calls mathematics "The nightlight on life's dark wall...." Besides, does a parody, or pastiche, really require a 100+-page discussion of convergent series and the like?

Will DFW's "real following" be upset to learn that the author may neither know nor "know" as much about trigonometric series as he appears to believe? Or that, by extension, his familiarity with optics, Quebecois dialect, and psychotropic drugs is not as extensive as the numerous footnotes to IJ suggest? Will they have any way of finding out?

The truth is probably less cunning than what Holt surmised. Wallace admitted on NPR's *Talk of the Nation*³² that "I got into this as an exercise in something called 'technical writing', which is a kind of nonfiction where you're trying to take really abstract or technical stuff and make it clear and pretty." I can live with DFW's lack of expertise in analysis and set theory. The relatively flat and repetitious, and for the most part neither clear nor pretty, writing in his "exercise" is another matter. The jacket of ASFTINDA quotes the *New York Times Book Review* calling DFW "a dynamic writer of extraordinary talent...he lays his artistic self on the line with his incendiary use of language, at times seeming to rip the mundane and the unusual from their moorings...." This is absolutely appropriate as a description of IJ or ASFTINDA, which I unfortunately have no reason to quote in this review. E&M's jacket and most reviews contain similar praise. But prosewise, the Mr. Chicken passage is as good as it gets. There are unusual metaphors, but they tend to be strained and unenlightening: in its cumulative nature, mathematics is "a towering baklava of abstractions and abstractions of abstractions" (p. 61), differential equations are compared to "integral calc on some sort of Class IV hallucinogen" (p. 151), Gödel is "modern math's absolute Prince of Darkness." The "exercise" may well be enough to satisfy hardcore Wallace fans who would not otherwise be motivated to read a book about any mathematical subject. That is arguably a good thing. It would be a shame, on the other hand, for a mathematically sophisticated reader to judge Wallace's style on the basis of E&M.

Most frustrating of all, though, is that the metaphysical concerns that fueled the first and best parts of the book get lost in DFW's attempt to make the technicalities "vivid and comprehensible." The author goes to great pains in the philosophical chapters to set a high standard for a solution of Zeno's paradoxes, considered relevant and philosophically problematic as recently as 2001.³³ He promises, quoting extensively from Bertrand Russell, that Cantor's creation of a consistent theory

of actual infinities is such a solution. But by the time Wallace gets to the theory, Zeno has vanished, no justification is given for the earlier promises, and the concluding discussion of the Continuum Hypothesis seems to suggest that in fact the solution has come at the cost of deeper metaphysical problems: "...a world with no finite circumference... spins, now, in a new kind of all-formal Void" (p. 303).

Readers of IJ may recognize here a familiar pattern: a problem having been posed as central at the beginning of the narrative, the reader is surprised to discover that the book ends with no resolution in sight. Discussions of IJ insist on the book's absence of "closure", one reviewer of E&M claiming that "Wallace is on record as designing *Infinite Jest* to resemble...a 'Sierpinski gasket',"³⁴ presumably because most of the crucial plot developments are left to the reader as an exercise. Frustrating the reader's expectations is arguably characteristic of postmodern fiction, and it is understandable that DFW would go to considerable lengths to keep his book from degenerating into the sort of passive "entertainment" on which much of IJ is an apocalyptic meditation and that the Great Discoveries series threatens to offer.

Maybe pop mathematics has to end this way. Still, I am used to a symphony ending with a cadence and mathematics with Q.E.D. We have already run across Bertrand Russell, in the role of DFW's star witness to the Greatness of Cantor's discovery. An active reader of E&M may be expected to seek out the sources of the Russell quotations. Most come from a particularly militant 1901 defense of Russell's formalist position, concluding with the disconcerting hope that "pure thought may achieve, within our generation, such results as will place our time, in this respect, on a level with the greatest age of Greece." The logical positivists who attempted to realize this vision turned their backs on metaphysics. Near the end of E&M, metaphysics makes a brief return in a footnote, in an evocation of the "Prince of Darkness" and his incompleteness theorem: "It's this shattering of the belief that 10% abstraction = 100% truth that pure math has still not recovered from—nor is it yet even clear what 'recovery' here would mean" (p. 302, note 121).

Does DFW intend this as an ironic retrospective comment on Russell's hopes for pure thought? Russell's article is probably best known for the assertion that "mathematics may be defined as the subject in which we never know what we are talking about nor whether what we are saying is true."³⁵ At the end of E&M the reader is left wondering whether this wasn't what Wallace meant all along.

Notes

1. Exactly seventeen on the Web according to Google.

2. In “Fear and loathing in globalization”, *New Left Review* 23 (September–October 2003).

3. R. Rucker, Infinite confusion, *Science* (January 16, 2004), 313a; A. Aczel, When good novelists do bad science, *The Globe and Mail* (Toronto) (January 3, 2004), D10.

4. The reviewer consulted five books about infinity for the general public, including E&M and Clegg’s book. The numbers in brackets indicate how many of the five discussed or alluded to the topic in question: the Greek term to *apeiron* for infinity [3], Pythagoras [5], the irrationality of $\sqrt{2}$ [5], and the fate of Hippasus [5]; Zeno’s paradoxes [5]; Aristotle and potential infinity [5]; Archimedes’ “sand reckoner” [3]; Augustine’s *City of God* [3]; Aquinas’s *Summa Theologica* [4]; Nicholas of Cusa [4]; Galileo’s *Two New Sciences* [5]; Cartesian coordinates [5]; Newton and Leibniz [5]; Berkeley’s attack on infinitesimals (“ghosts of departed quantities”) [3]; Gauss’s refusal to countenance actual infinities [5]; Bolzano’s *Paradoxes of the Infinite* [5] and his pacifism [3]; the Riemann sphere (with its point at infinity) [3]; Weierstrass’s skill at drinking and fencing [3]; the transcendence of π [5]; Dedekind cuts [4]; Kronecker’s rejection of infinity and his persecution of Cantor [4]; Cantor’s theory of ordinals [4], his proof of the countability of \mathbb{Q} [5], his diagonalization argument [5], “je le vois, mais je ne le crois pas” (written by Cantor, in French, in a letter to Dedekind concerning his proof of the commensurability of the line and the plane) [5], and the continuum hypothesis [5]; Peano’s set-theoretic definition of the integers [5]; Russell’s paradox [5]; Hilbert’s Hotel [4]; Gödel’s incompleteness theorem [5] and his starvation [5]; and Cohen’s proof of independence of the continuum hypothesis [5].

5. According to <http://www.publishingtrends.com/copy/03/0302/0302bookview.html>, whose reliability I have no way to verify, one backer of Atlas Books is Roger Altman, former deputy Treasury secretary, now publisher of the *National Enquirer* and *Weekly World News*.

6. Bookseller.com, October 9, 2003; see <http://www.thebookseller.com/?pid=275&did=9125>. Atlas Books is located in the HarperCollins Building on 53rd Street in Manhattan. HarperCollins itself is a subsidiary of Rupert Murdoch’s News Corp., along with Fox News, the *New York Post*, and the *Times Literary Supplement*.

7. http://www.publishingnews.co.uk/pn/pno_ff2003news33.asp.

8. From W. W. Norton press material accompanying my review copy of E&M, featuring advance praise by James Gleick, *New York Magazine*, and *Playboy*.

9. Quoted by Dalia Sofer in “The narrative approach to science”, *Poets and Writers Magazine* online, <http://www.pw.org/mag/0309/newssofer2.htm>.

10. See last note.

11. Ubiquitous in E&M is IYI (If You’re Interested), supposedly allowing DFW to pack at least two books into E&M, designed for readers with different degrees of mathematical knowledge. Some other random examples: D.B.P. (Divine Brotherhood of Pythagoras), V.I.R. (Vicious Infinite Regress), G.C.P.F.S. (General Convergence Problem of Fourier Series, used only once as far as I can tell).

12. Like most *Notices* readers, Wallace has spent nearly all his adult life as a professional academic.

13. An example: “The father of abstraction in mathematics: Pythagoras. The father of abstraction in philosophy: Plato.” (E&M, p. 10).

14. The first time I have seen this word in a book of any kind about mathematics or, for that matter, in the *AMS Notices*.

15. There are 408 footnotes in E&M, numbered separately by chapter. The endnotes to IJ, some of them with their own footnotes, contain crucial plot information and run to 97 pages, or 8.99% of the total volume of the book.

16. Caleb Crain, *Boston Globe*, October 26, 2003.

17. Building on the results of Gödel and Turing—soon to be featured in the Great Discoveries series, by the way—on undecidability and his own work on algorithmic complexity, Gregory Chaitin has argued vigorously in several books for general readers that the distinction between knowing and “knowing” is still operative where the continuum is concerned. The French philosophers Jean Cavailles and Jean-Toussaint Desanti wrote long and difficult books about the possibility of phenomenological knowledge (as opposed to “knowledge”) of the continuum, given that purely logical knowledge is apparently excluded. In *Infinity and the Mind* (1982), on the other hand, Rudy Rucker proposes a number of ingenious ways finite beings like us might really know, and not merely “know”, infinity but does not go so far as to argue that we could really know that we really know.

18. Augustine, in *City of God*, quoted in R. Rucker, op. cit., pp. 3–4; Aquinas, in *Summa Theologiae*, quoted in E&M, p. 93; Hume, in *A Treatise on Human Nature*, quoted by B. Clegg, op. cit., p. 219.

19. “Dies sind die beiden gewichtigsten Gründe, welche im Laufe der Zeiten gegen das Transfinitum vorgeführt worden sind...”, G. Cantor, *Mitteilungen zur Lehre vom Transfiniten*, in *Gesammelte Abhandlungen* (Berlin: Springer-Verlag, 1932), pp. 378–439. The quotation is on p. 404, and DFW’s recollection on p. 93 of E&M is not altogether accurate. Cantor considers his theory a response to two of Aquinas’s arguments: one concerning enumeration, the second the “clear intention of the Creator.” Perhaps understandably, DFW omitted to recall the latter. He also translates Aquinas’s *multitudo* as “set”, which looks anachronistic, though it may be a reasonable equivalent of Cantor’s “Menge”; I am in no position to say.

20. Disclaimer: I am not competent to gauge the accuracy of DFW’s account of Greek philosophy, and the last footnote suggests caution. I am pretty confident of this judgment nonetheless.

21. This curious use of the word “sexy”, which recurs several times in E&M, should not be seen as a bit of unexpected good news for *Notices* readers who teach calculus; apparently it reflects a purely linguistic evolution rather than a significant cultural shift.

22. Trivial, perhaps, but not stupid; according to Max Black, “This kind of mathematical solution has behind it the authority of Descartes and [C. S.] Pierce and Whitehead,” among others, though Black agrees that it does not “go to the heart of the matter.” See Black’s “Achilles and the tortoise”, reprinted in *Zeno’s Paradoxes* (W. Salmon, ed.), Hackett, 2001, p. 70.

23. The first part is from p. 43, the second a quotation on p. 42 from Hardy’s *A Mathematician’s Apology*.

24. Near the end of the 200 pages, DFW admits that “All you really need to get is a rough sense of how the Uniqueness Theorem leads Cantor into transfinite math” (p. 233, note 14).

25. In Rucker’s review in *Science*; see note 3.

26. On p. 44 of his *Fourier Series* (Springer-Verlag Graduate Texts in Mathematics, 1979), R. A. Edwards refers to "Dogged insistence on the pointwise interpretation of convergence."

27. "It's the hardest part of the book." Wallace, quoted in Crain, op. cit. On several occasions Wallace insists on the distinction between Fourier series and general trigonometric series, without making clear how they differ beyond saying that the former have Fourier coefficients, which are "so conceptually hairy that we plan to avoid them at almost any cost" (p. 116).

28. David Papineau, *New York Times Book Review*, November 16, 2003.

29. Had Atlas and Norton seen fit to send the manuscript to a mathematical reader before publication, most of these blunders could easily have been avoided, though occasionally at the cost of substantial rewriting. I note in this connection recurrent nervous allusions in E&M to unspecified time and space limitations.

30. See Stephen Burn.

31. "E Unibus Pluram," in ASFTINDA.

32. On October 13, 2003. "I wince at the idea that there are mathematicians in the audience," said Wallace, "because I'm by no means an expert."

33. See Wesley C. Salmon, *Zeno's Paradoxes*, for important twentieth-century contributions to an understanding of Zeno's paradoxes and an extensive bibliography of relevant articles, (mainly) in English, current through 2001. For echoes in continental philosophy, see V. Tasić *Mathematics and the Roots of Postmodern Thought*, and my review in the August 2003 issue of the *Notices*.

34. Review by D. Kipen in the *San Francisco Chronicle*, November 18, 2003. Why not a Cantor set, I wonder?

35. B. Russell, *Mathematics and the metaphysicians*, from *Mysticism and Logic* (London: Longmans Green, 1918), pp. 74-96; reprinted in James R. Newman, *The World of Mathematics* (1956), quotations from p. 1590 and p. 1577. The article was originally published in 1901 as "Recent work on the principles of mathematics", *International Monthly* 4, 83-101.

Coming Summer 2004

Discrete Algorithmic Mathematics

Third Edition

Stephen B Maurer Anthony Ralston

ISBN 1-56881-166-7

Approx. 800 pp.

Hardcover

\$74.00 (tent.)

Thoroughly revised for a one-semester course, this well-known and highly regarded book is an outstanding text for undergraduate discrete mathematics.

New or extended discussions of:

- order notation
- generating functions
- chaos
- aspects of statistics
- computational biology

Written in a lively, clear style that talks to the reader, the book is unique for its emphasis on *algorithmics* and the *inductive and recursive paradigms* as central mathematical themes. It includes a broad variety of applications, not just to mathematics and computer science, but to natural and social science as well.

A complete solution manual will be available to instructors.

For more information, and to request an examination copy, visit www.akpeters.com/dam3.



A K Peters, Ltd.
888 Worcester St. Suite 230
Wellesley, MA 02482
Tel: 781.416.2888 Fax: 781.416.2889