1. (a) $(A^c \cup A^c)^c \cup (A^c \cap A) = (A^c)^c \cup \emptyset = A \cup \emptyset = A.$

$(A \cap B) \cup (A \setminus B) = A.$

(b) Let $x \in X$. Then

$$x \in A \setminus (B \cap C) \iff x \in A \text{ and } x \notin B \cap C$$

$$\overset{\text{de Morgan}}{\iff} x \in A \text{ and either } x \notin B \text{ or } x \notin C$$

$$\overset{\wedge \text{ distributes over } \vee}{\iff} \text{Either } x \in A, x \notin B \text{ or } x \in A, x \notin C$$

$$\iff x \in (A \setminus B) \cup (A \setminus C)$$

2. (a) $\emptyset, \{2\}, \{3\}, \{5\}, \{2,3\}, \{2,5\}, \{3,5\}, \{2,3,5\}$

4 contain an even number.

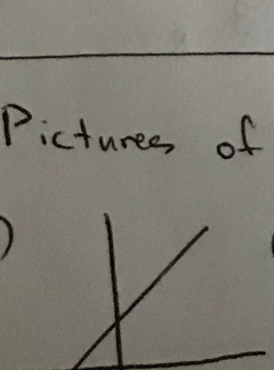
Note: $\emptyset \neq \{\emptyset\}$.

(b) # at most 3 elt subsets = # 0 elt subsets + # 1 elt subsets + ... + # 3 elt subsets

$$= \binom{10}{0} + \binom{10}{1} + \binom{10}{2} + \binom{10}{3} = 176.$$

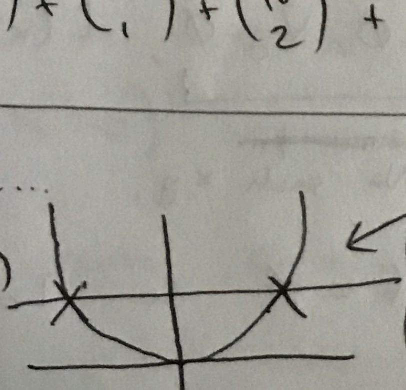
3. Pictures of graphs are fine...

(a)



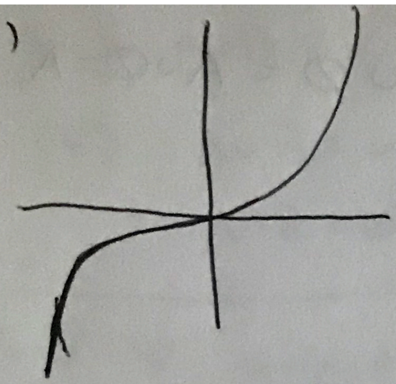
Bijective

(b)



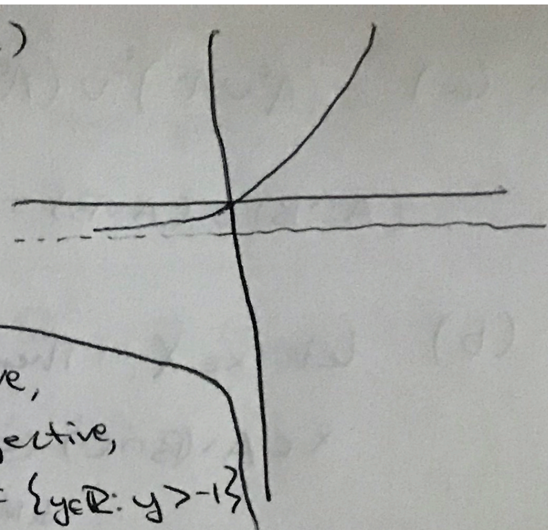
Not injective, not surjective, image $= \{x \in \mathbb{R} : x \geq 0\}$

(c)



(d)

Bijective



Injective,
not surjective,
image $= \{y \in \mathbb{R} : y > -1\}$

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4. <u>Uniqueness</u>: A function $\emptyset \longrightarrow S$ is a special type of subset of $\emptyset \times S = \emptyset$, and there is only one subset of $\emptyset$, ~~so there~~ namely $\emptyset$.

Call this relation f. We must show:

<u>Existence</u>: f is a function: Have to show:

$$(\forall x \in \emptyset)(\exists! y \in S): f(x) = y.$$

This is vacuously true. ✓

<u>Claim</u>. f is injective. f is surjective iff $S = \emptyset$.

<u>Proof</u>. f is vacuously injective:

$$\underbrace{\forall x \in \emptyset, \forall y \in \emptyset}_{\text{No such } x, y.}, \ f(x) = f(y) \Rightarrow x = y.$$

Also, $\operatorname{Im} f = \emptyset$ so $\operatorname{Im} f = S \iff S = \emptyset$.

□

<u>Bonus</u>: <u>False</u>: "Colorado has Ocean beaches, and they all have blue sand"

<u>True</u>: "Every Ocean beach in Colorado has blue sand."

5. (a) $A \cup B = \mathbb{R}^2$, $A \cap B =$ the line $y=x$.

(b) $A \cup B =$ complement of the line $y=x$,

$A \cap B = \emptyset$.

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6. (a) φ surjective $\iff \forall (e,f) \in \mathbb{R}^2,\ \exists\ (x,y) \in \mathbb{R}^2$ s.t.

$$\varphi(x,y) = (e,f).$$

But $\varphi(x,y) = (L_1(x,y),\ L_2(x,y))$,

So this true iff the system

$$L_1(x,y) = e$$
$$L_2(x,y) = f$$

has a solution $\forall e,f$.

(b) From linear algebra,

$$\varphi \text{ is surjective} \iff \operatorname{rank}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = 2$$

$$\overset{\text{rank}+\text{nullity}=2}{\iff} \operatorname{nullity}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = 0$$

$$\iff \varphi \text{ is injective}.$$

$(\iff ad-bc=0)$

(c) $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ works since $ad-bc=0$.

7. (a) $4^2 = 16$ from B to A

$2^4 = 16$ from A to B.

(b) 4 choices of $f(a)$, 3 of $f(b)$ so should get 12:

$f(a) = a_1$, $f(b) = a_2$	$f(a) = a_2$, $f(b) = a_1$
— " — $f(b) = b_1$	— " — $f(b) = b_1$
— " — $f(b) = b_2$	— " — $f(b) = b_2$
$f(a) = b_1$, $f(b) = a_1$	$f(a) = b_2$, $f(b) = a_1$
— " — $f(b) = a_2$	— " — $f(b) = a_2$
— " — $f(b) = b_2$	— " — $f(b) = b_1$

2 non surjective from A to B:

(1) $f(a_1) = f(a_2) = f(b_1) = f(b_2) = a$

(2) $f(a_1) = f(a_2) = f(b_1) = f(b_2) = b$.