MODERN ALGEBRA I GU4041

Homework 5, due February 27: Permutations

1. Judson, Section 5.3, exercise 1 and 2 (a)-(f), 2(j).

2. Let Σ_n denote the group of permutations of n letters. What is the maximal order of an element of Σ_4? Write down an element of Σ_4 with the maximal order and decompose it as a product of disjoint cycles.

3. Find two permutations of 4 letters σ and τ such that $\sigma^2 = \tau^2 = e$ but $\sigma\tau \neq \tau\sigma$.

4. Label the corners of a square $1, 2, 3, 4$ as in the diagram. Let $D \subset \Sigma_4$ be the set of permutations of the corners that preserve the square. Show that D is a subgroup of Σ_4. What is its order?

5. What are the orders of the following permutations?
(a) (132) in Σ_3 (b) $(156)(234)$ in Σ_6 (c) $(14235)^2$ in Σ_5.

Recommended reading

Judson book, Section 5.1; Howie's notes, Chapter 4.