## MODERN ALGEBRA I GU4041

## Homework 13, due May 5: Groups of small order

1. Let $p>3$ be a prime number. Show that any group of order $3 p$ is solvable.
2. Judson, section 14.4, exercises 11,12 .
3. Show that no group of order 64 or 96 is simple. Construct two distinct non-abelian groups of each order.
4. Judson, section 15.3, exercises 20, 22, 23.
5. Show that no group of order 112 is simple. (Hint: if the group $G$ is simple then it admits an injective homomorphism to the symmetric group $S_{r}$, where $r$ is the number of 2-Sylow subgroups.)

## Recommended Reading

Gallagher notes 22, 23, 24.

