MODERN ALGEBRA I GU4041

Homework 12, due April 30: Sylow theorems

1. Let p be an odd prime number. Show that any group of order $2p$ is either cyclic or isomorphic to D_{2p}.

2. Let A be a finite abelian group of order N. Let $p_1 < p_2 < \cdots < p_n$ denote the distinct prime numbers dividing N.

(a) Prove that A has a unique Sylow p-subgroup A_i of order **a power of** p_i for $i = 1, \ldots, n$.

(b) Show that

$$A \xrightarrow{\sim} A_1 \times A_2 \times \cdots \times A_n.$$

3. Construct p-Sylow subgroups of the symmetric group S_5 and the alternating group A_5 for $p = 2, 3, 5$.

4. Judson, section 15.3, exercises 1, 3, 6, 7, 9.

Recommended reading

Gallagher notes 18, 19; Judson, Chapter 15.