
2. Howie notes, section 6.6, exercises 1 and 2.

3. Choose a subgroup $H$ of order 2 in $S_3$.
   (a) Find $g \in S_3$ such that $gHg^{-1} \neq H$, thus demonstrating that $H$ is not a normal subgroup.
   (b) Write down representatives of the sets of left cosets $S_3/H$ and right cosets $H\backslash S_3$.

4. (from the Judson book, section 6.4, exercise 11): Let $H$ be a subgroup of a group $G$ and let $g_1, g_2 \in G$. Show that the following are equivalent:
   - $g_1H = g_2H$
   - $Hg_1^{-1} = Hg_2^{-1}$
   - $g_1H \subset g_2H$
   - $g_1 \in g_2H$
   - $g_1^{-1}g_2 \in H$

5. Let $G$ denote the set of $3 \times 3$ matrices with entries in $\mathbb{R}$, of the form
   \[
   \begin{pmatrix}
   a & b & e \\
   c & d & f \\
   0 & 0 & \lambda
   \end{pmatrix}
   \]
   that satisfy the relation
   \[(ad - bc)\lambda = 1.\]
   (a) Show that $G$ is a group.
   (b) Show that the subset $H \subset G$ for which $a = d = 1$ and $b = c = 0$ is a subgroup.
   (c) Show that $H$ is a normal subgroup of $G$.
   (d) Let $\phi : G \to GL(2, \mathbb{R})$ be the map
   \[
   \phi\left(\begin{pmatrix}
   a & b & e \\
   c & d & f \\
   0 & 0 & \lambda
   \end{pmatrix}\right) = \begin{pmatrix}
   a & b \\
   c & d
   \end{pmatrix}.
   \]
   Show that $\phi$ is a homomorphism and that $\phi(g)$ is the identity matrix if and only if $g \in H$. 

6. Let $n > 2$ be an integer. Show that the group of rotations of the regular $n$-gon is a normal subgroup of the dihedral group $D_{2n}$, and identify the quotient group.

**Recommended reading**

Howie notes, section 3.4, sections 6.1-6.3