ALGEBRAIC NUMBER THEORY W4043

Homework, week 11, due December 6

I. Hindry's book, Exercises 6.1, p. 158, and 6.4 and 6.6, p. 159.

II. Let K be a number field, $Cl(K)$ the ideal class group, $\chi : Cl(K) \rightarrow \mathbb{C}^{\times}$ a homomorphism. If $\mathfrak{a} \subset \mathcal{O}_K$ is any ideal, let $[\mathfrak{a}]$ denote its ideal class in $Cl(K)$, and define $\chi(\mathfrak{a}) = \chi([\mathfrak{a}])$.

1. Show that $|\chi(\mathfrak{a})| = 1$ for any ideal $\mathfrak{a}$.

2. Let

$$L(s, \chi) = \sum_{\mathfrak{a} \subset \mathcal{O}_K} \frac{\chi(\mathfrak{a})}{N\mathfrak{a}^s}.$$

Show that $L(s, \chi)$ is absolutely convergent for $Re(s) > 1$. What would you need to know in order to prove that $L(s, \chi)$ converges for $Re(s) > 0$?

3. Let K be a number field, S a finite set of prime ideals of $\mathcal{O}_K$. Let I^S denote the set of ideals of $\mathcal{O}_K$ not divisible by any prime ideal of S. Let χ be as above. Define

$$L^S(s, \chi) = \sum_{\mathfrak{a} \in I^S} \frac{\chi(\mathfrak{a})}{N\mathfrak{a}^s}.$$

Express $L^S(s, \chi))$ in terms of $L(s, \chi)$ and the set S. Does the function $L^S(s, \chi)$ have any obvious zeroes?