

ALGEBRAIC NUMBER THEORY W4043

HOMEWORK, WEEK 10, DUE DECEMBER 3

1. Let f be the polynomial

$$f(X) = (X^2 - 13)(X^2 - 17)(X^2 - 221).$$

(a) Show that the equation $f(X) = 0$ has no solutions in \mathbb{Q} but that it has solutions in \mathbb{R} .

(b). Show that 17 is a square modulo 13. Show that $f(X) = 0$ has a solution in the 13-adic ring \mathbb{Z}_{13} .

(c) Show that $f(X) = 0$ has a solution in \mathbb{Z}_{17} .

(d) Show that $f(X) = 0$ has a solution in \mathbb{Z}_2 .

(e) Show that for every prime $p \neq 2, 13, 17$, $f(X) = 0$ has a solution in \mathbb{Z}_p .

Thus $f(X) = 0$ has a solution modulo N for all integers N , and a solution in \mathbb{R} , but no rational solution.

2. Hindry's book, p. 124, Exercise 6.24.