CALCULUS III, UN1201, SECTIONS 6 AND 7

Practice Midterm 2

No electronic devices (laptops, calculators, telephones) or notes are allowed during exams. Numerical expressions involving square roots, trigonometric functions, or inverse trigonometric functions need not be simplified and should not be converted into decimals.

Show all work. To receive full credit, you must justify your answers.

Use the back side of the page if you need more space to do a problem.

1	2	3	4	5

Total:

Some possibly useful formulas

The curvature of the curve given by the vector function $\vec{r}$ is

$$\kappa(t) = \frac{|\vec{r}'(t) \times \vec{r}''(t)|}{|\vec{r}'(t)|^3}.$$

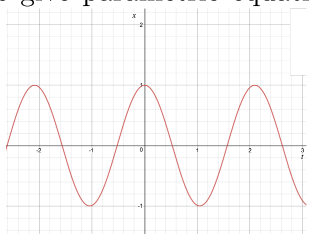
If $\vec{r}$ is given by $\vec{r}(t) = \langle t, f(t), 0 \rangle$ for some function f, so the curve it parametrizes is the graph of the function $y = f(x)$ in the plane, then the curvature at the point $(x, f(x), 0)$ is

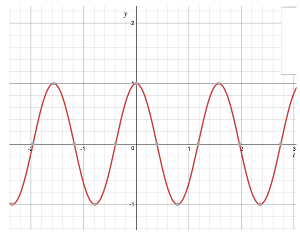
$$\kappa(x) = \frac{|f''(x)|}{[1 + (f'(x))^2]^{3/2}}.$$

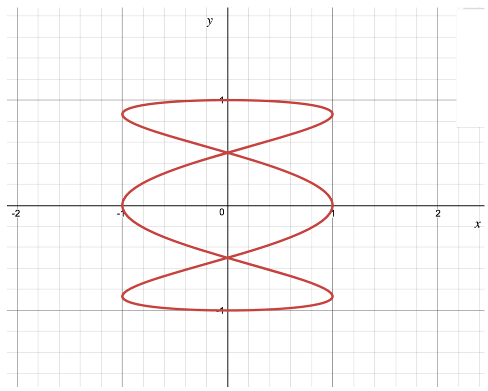
1. (10 points) (a) True or false? 5 points for each correct answer **with correct justification**, 0 points for wrong answer, 2 points for "don't know."

(i) If the graph of a function $z = f(x, y)$ is a plane, then all the second partial derivatives of f equal 0.

(ii) The graphs on the left of the functions $x = x(t)$ and $y = y(t)$ on the first line give parametric equations of the curve on the following line.







1. (b) (10 points)

(i) Give an example of a function $f(x, y)$ on the domain $-1 \leq x \leq 1$ and $-1 \leq y \leq 1$ for which $\frac{\partial f}{\partial x}$ is continuous for all (x, y) but for which $\frac{\partial f}{\partial y}$ does not exist at $(x, y) = (0, 0)$.

(ii) Give an example of a parametric curve $\vec{r}(t)$ for $-1 \leq t \leq 1$ such that the curvature $\kappa(t)$ equals 0 for $t = 0$ but $\kappa(t) \neq 0$ if $|t| \neq 0$.

2. (a) Find the unit tangent vector, unit normal vector, and unit binormal vector to the parametric curve

$$\vec{t} = sin^2(t)\mathbf{i} + \sin^3(t)\mathbf{j} + \cos^3(t)\mathbf{k}$$

on the open interval $[0, \frac{\pi}{2}]$.

(b) Using the formula $\sin(2\theta) = 2\cos(\theta)\sin(\theta)$, compute the length of the curve.

3. (a) Find a parametrization of the intersection of the cylinders $x = \frac{y^2}{4}$ and $z = \ln(x)$ on the interval $[\frac{1}{2}, 2]$ and compute the curvature as a function of the parameter. Write down the formula for the length of the curve as a definite integral.

(b) Explain how to find the arclength parametrization for the curve above; you do not need to solve the equation.

(c) Find the equation of the osculating plane to the parametric curve

$$\mathbf{r}(t) = t\mathbf{i} + t\cos(t)\mathbf{j} + t\sin(t)\mathbf{k}$$

at the point $P = (\pi, \pi, 0)$.

4. (a) Which of these limits exists?

(i) $\lim_{(x,y)\to(0,0)} \frac{xy^2cos(y)}{x^2+y^4}$

(ii) $\lim_{(x,y,z)\to(0,0,0)} \frac{(x^2+y^2+1)(cos(z)-1)}{(x^2-y^2+2)(z^2)}$

(iii) Let $f(x,y) = \frac{(xy)^p}{x^4+y^4}$ if $(x,y) \neq (0,0)$ for some integer p. Show that $\lim_{(x,y)\to(0,0)} f(x,y)$ exists if $p > 2$ but not if $p = 1$ or $p = 2$.

(b) Determine the level surfaces of the function

$$f(x,y,z) = x + y^2 + z^2.$$

(c) Let $g(x,y,z) = (x + y^2 + z^2) \cdot e^{y^2+z}$. Compute the partial derivatives f_x, f_y, f_z, and $\frac{\partial^2 f}{\partial x \partial z}$.

5. (a) Find the equation of the tangent plane to the surface $x^2 - 2xy + 7z^2 = 15$ at the point $(4, 1, -1)$.

(b) Find symmetric equations for the line passing through the point $(4, 1, -1)$ and perpendicular to the tangent plane found in (a).

(c) The interior volume of a submarine of height x, length y, and width z is approximately $4((x - .5)yz) + \frac{2}{3}(yz)^2$. Suppose the dimensions (in meters) are approximately $x = 4$, $y = 10$, $z = 3$, with possible error .01 meter in each direction. In order to provide enough oxygen for the crew, one requires 100 cubic meters per person. Use a linear approximation to determine whether or not there is enough oxygen for 6 people.