CALCULUS III, UN1201, SECTIONS 6 AND 7

Practice Midterm 1, October 2018

No electronic devices (laptops, calculators, telephones) or notes are allowed during exams. Numerical expressions involving square roots, trigonometric functions, or inverse trigonometric functions need not be simplified and should not be converted into decimals.

To receive full credit on the actual midterm, you must show all work and justify your answers.

1. True or false? 5 points for each correct answer, 0 points for wrong answer, 2 points for "don't know."

(a) Suppose $\vec{u}, \vec{v}$, and $\vec{w}$ are vectors in 3-space. Then

$$(\vec{u} \times \vec{v}) \times \vec{w} = \vec{u} \times (\vec{v} \times \vec{w}).$$

(b) If three points P, Q, R in 3-space are collinear, then there is more than one plane containing P, Q, and R.

(c) Let $P = (5, 4, 7)$, $Q = (8, 4, 4)$, $R = (4, 2, 3)$. All angles in the triangle with vertices P, Q, R are acute (less than $\frac{\pi}{2}$).

(d) Suppose the point $P = (x, y, z)$ lies on the quadric surface $\frac{z^2}{4} - x^2 - y^2 = 3$. Then either $z > 2\sqrt{x^2 + y^2}$ or $-z < -2\sqrt{x^2 + y^2}$.

2. (a) Let $P = (5, -4, 7)$, $Q = (8, 4, 7)$, $R = (-4, 2, 2)$. Show that the three points are not collinear and find an equation for the plane $\mathcal{P}$ containing them.

(b) Let $\vec{u} = \langle -1, -1, 2 \rangle$. Find the orthogonal projection of $\vec{PQ}$ on $\vec{u}$.

(c) Write down the two unit normal vectors to $\mathcal{P}$. Let $\vec{n}$ be one of them, and find the parametric equations of the line through R parallel to $\vec{n}$.

3. (a) Write the equation of the plane $2x - y + 3z = 10$ in cylindrical coordinates.

(b) A quadric surface has the equation $cos^2(\varphi) = cos(2\theta)sin^2(\varphi)$. What kind of surface is it?

4. For each equation below, sketch the traces in the indicated boxes, making the axes clear (but not the vertices). Identify the surface; you may include a small sketch of the surface if you prefer. (In the midterm there will be 3 such equations, not 5)

(a) $x^2 + 10y^2 + 4x^2 = 10$.

xy trace	xz trace	yz trace	Type of surface

(b) $z = 15y^2 - 12x^2 + 10$.

xy trace	xz trace	yz trace	Type of surface

(c) $\frac{x^2}{5} = 4y^2 - \frac{z^2}{3} - 1$.

xy trace	xz trace	yz trace	Type of surface

(d) $\frac{y^2}{2} - \frac{z^2}{7} - x^2 = 1$

xy trace	xz trace	yz trace	Type of surface

(e) $z^2 - \frac{y^2}{4} - \frac{x^2}{9} = 0$.

xy trace	xz trace	yz trace	Type of surface

5. (a) Let $\mathcal{P}_1$ be the plane with equation $x - y + 2z = 4$ and let $\mathcal{P}_2$ be the plane with equation $2x + y - z = 2$. Let ℓ be the line in which $\mathcal{P}_1$ and $\mathcal{P}_2$ intersect. Find parametric equations of ℓ.

(b) Find the equation of the plane $\mathcal{P}_3$ that is parallel to $\mathcal{P}_1$ and passes through the point $Q = (0, 1, 0)$. Show that the point $R = (3, 0, -2)$ lies on $\mathcal{P}_3$.

(c) Let m be the line through the origin and the point R. Find the intersections of m with the planes $\mathcal{P}_1$ and $\mathcal{P}_2$.

(d) Find the points A on ℓ and B on m that are closest to each other, and determine the distance between ℓ and m. Find the symmetric equations for the line that is perpendicular to both ℓ and m and passes through A and B.