REPRESENTATION THEORY W4044

1. Homework, week 6, due March 4

1. James and Liebeck book, Chapter 15, Exercises 1, 2, 3; Chapter 16, Exercises 4, 5.

2. Let G be any finite group with n elements, and consider the right regular representation on $V = \mathbb{C}[G]$.

(a) Suppose G is abelian. How many different non-zero projectors are there from V to itself that are invariant with respect to the G action?

(b) In general, suppose there are only finitely many distinct non-zero projectors from V to itself, invariant with respect to the G action. Show that G is abelian.

3. (a) Let (ρ, V) be a representation of the group G, with character χ. Let

$$N = \{g \in G \mid \chi(g) = \chi(1)\}.$$

Show that N is a normal subgroup of G.

(b) Suppose that for every non-trivial character of G and every element $g \neq e$ we have $\chi(g) \neq \chi(e)$. Show that G has no normal subgroups.

4. Let $A_4 \subset S_4$ be the alternating group, the subgroup of index 2 consisting of even permutations. Let (ρ, V) be the representation of S_4 on $\mathbb{C}^4/\mathbb{C}(1,1,1,1)$, and consider ρ as a representation of the subgroup A_4. (a) Determine the conjugacy classes in A_4.

(b) Compute the character of ρ and show that ρ is an irreducible representation of A_4.

(c) Deduce that A_4 has three distinct 1-dimensional representations.