

## MODERN ALGEBRA II W4042

HOMEWORK, WEEK 8, DUE NOVEMBER 5

- (a) What is the degree of  $\mathbb{Q}(\sqrt{2} + 3i)$  as an extension of  $\mathbb{Q}$ ?

(b) Find the minimal monic polynomial of  $\sqrt{2} + 3i$  in  $\mathbb{Q}[X]$ .
- Let  $K$  be a field and let  $R$  be a commutative ring containing  $K$  that is also a finite-dimensional  $K$ -vector space. Suppose  $R$  is an integral domain. Prove that  $R$  is a field.
- Let  $L \supset K$  be a finite extension of fields, with  $[L : K] = p$  an odd prime number.

(a) List all the subfields  $K' \subset L$  containing  $K$ .

(b) Suppose  $L = K(\alpha)$  for some  $\alpha \in L$ . Let  $K' = K(\alpha^2) \subset L$ . What are the possible degrees of  $L$  over  $K'$ ?
- Rotman's book, Exercises 72 (iii), 73, and 77, p. 58.