4. The projection of \((2, 3, 5)\) onto the \(xy\)-plane is \((2, 3, 0)\); onto the \(yz\)-plane, \((0, 3, 5)\); onto the \(xz\)-plane, \((2, 0, 5)\).

The length of the diagonal of the box is the distance between the origin and \((2, 3, 5)\), given by

\[
\sqrt{(2 - 0)^2 + (3 - 0)^2 + (5 - 0)^2} = \sqrt{38} \approx 6.16
\]
6. In $\mathbb{R}^3$, the equation $y = 3$ represents a vertical plane that is parallel to the $xz$-plane and 3 units to the right of it. The equation $z = 5$ represents a horizontal plane parallel to the $xy$-plane and 5 units above it. The pair of equations $y = 3, z = 5$ represents the set of points that are simultaneously on both planes, or in other words, the line of intersection of the planes $y = 3, z = 5$. [continued]
This line can also be described as the set \( \{(x, 3, 5) \mid x \in \mathbb{R}\} \), which is the set of all points in \( \mathbb{R}^3 \) whose \( x \)-coordinate may vary but whose \( y \)- and \( z \)-coordinates are fixed at 3 and 5, respectively. Thus the line is parallel to the \( x \)-axis and intersects the \( yz \)-plane in the point \((0, 3, 5)\).
20. Completing squares in the equation $3x^2 + 3y^2 - 6y + 3z^2 - 12z = 10$ gives

$$3x^2 + 3(y^2 - 2y + 1) + 3(z^2 - 4z + 4) = 10 + 3 + 12 \implies 3x^2 + 3(y - 1)^2 + 3(z - 2)^2 = 25 \implies$$

$$x^2 + (y - 1)^2 + (z - 2)^2 = \frac{25}{3},$$

which we recognize as an equation of a sphere with center $(0, 1, 2)$ and radius $\sqrt{\frac{25}{3}} = \frac{5}{\sqrt{3}}$. 
4. (a) The initial point of $\overrightarrow{BC}$ is positioned at the terminal point of $\overrightarrow{AB}$, so by the Triangle Law the sum $\overrightarrow{AB} + \overrightarrow{BC}$ is the vector with initial point $A$ and terminal point $C$, namely $\overrightarrow{AC}$.

(b) By the Triangle Law, $\overrightarrow{CD} + \overrightarrow{DB}$ is the vector with initial point $C$ and terminal point $B$, namely $\overrightarrow{CB}$.

(c) First we consider $\overrightarrow{DB} - \overrightarrow{AB}$ as $\overrightarrow{DB} + (\overrightarrow{-AB})$. Then since $\overrightarrow{-AB}$ has the same length as $\overrightarrow{AB}$ but points in the opposite direction, we have $\overrightarrow{-AB} = \overrightarrow{BA}$ and so $\overrightarrow{DB} - \overrightarrow{AB} = \overrightarrow{DB} + \overrightarrow{BA} = \overrightarrow{DA}$.

(d) We use the Triangle Law twice: $\overrightarrow{DC} + \overrightarrow{CA} + \overrightarrow{AB} = (\overrightarrow{DC} + \overrightarrow{CA}) + \overrightarrow{AB} = \overrightarrow{DA} + \overrightarrow{AB} = \overrightarrow{DB}$. 
8. We are given \( \mathbf{u} + \mathbf{v} + \mathbf{w} = \mathbf{0} \), so \( \mathbf{w} = (-\mathbf{u}) + (-\mathbf{v}) \). (See the figure.)

Vectors \(-\mathbf{u}\), \(-\mathbf{v}\), and \(\mathbf{w}\) form a right triangle, so from the Pythagorean Theorem we have \( |\mathbf{u}|^2 + |\mathbf{v}|^2 = |\mathbf{w}|^2 \). But \( |\mathbf{u}| = |\mathbf{u}| = 1 \) and \( |\mathbf{v}| = |\mathbf{v}| = 1 \) so \( |\mathbf{w}| = \sqrt{|\mathbf{u}|^2 + |\mathbf{v}|^2} = \sqrt{2} \).
22. \[ a + b = \langle 8 + 5, 1 + (-2), -4 + 1 \rangle = \langle 13, -1, -3 \rangle \]

\[
4a + 2b = 4 \langle 8, 1, -4 \rangle + 2 \langle 5, -2, 1 \rangle = \langle 32, 4, -16 \rangle + \langle 10, -4, 2 \rangle = \langle 42, 0, -14 \rangle
\]

\[
|a| = \sqrt{8^2 + 1^2 + (-4)^2} = \sqrt{81} = 9
\]

\[
|a - b| = |\langle 8 - 5, 1 - (-2), -4 - 1 \rangle| = |\langle 3, 3, -5 \rangle| = \sqrt{3^2 + 3^2 + (-5)^2} = \sqrt{43}
\]
6. (a) \(x = \sqrt{3}\) and \(y = -1\) \(\Rightarrow\) \(r = \sqrt{(\sqrt{3})^2 + (-1)^2} = 2\) and \(\tan \theta = \frac{-1}{\sqrt{3}}\) \([\theta = -\frac{\pi}{6} + n\pi]\). Since \((\sqrt{3}, -1)\) is in the fourth quadrant, the polar coordinates are (i) \((2, \frac{11\pi}{6})\) and (ii) \((-2, \frac{5\pi}{6})\).

(b) \(x = -6\) and \(y = 0\) \(\Rightarrow\) \(r = \sqrt{(-6)^2 + 0^2} = 6\) and \(\tan \theta = \frac{0}{-6} = 0\) \([\theta = n\pi]\). Since \((-6, 0)\) is on the negative \(x\)-axis, the polar coordinates are (i) \((6, \pi)\) and (ii) \((-6, 0)\).
2. (a) \[ (\sqrt{2}, \frac{3\pi}{4}, 2) \]

\[
x = \sqrt{2} \cos \frac{3\pi}{4} = \sqrt{2} \left(-\frac{\sqrt{2}}{2}\right) = -1,
\]

\[
y = \sqrt{2} \sin \frac{3\pi}{4} = \sqrt{2} \left(\frac{\sqrt{2}}{2}\right) = 1,
\]

so the point is \((-1, 1, 2)\) in rectangular coordinates.
(b)

\[ x = 1 \cos 1 = \cos 1, \quad y = 1 \sin 1 = \sin 1, \quad \text{and} \quad z = 1, \]

so the point is \((\cos 1, \sin 1, 1) \approx (0.54, 0.84, 1)\) in rectangular coordinates.
6. Since $\theta = \frac{\pi}{6}$ but $r$ and $z$ may vary, the surface is a vertical plane including the $z$-axis and intersecting the $xy$-plane in the line $y = \frac{1}{\sqrt{3}} x$. (Here we are assuming that $r$ can be negative; if we restrict $r \geq 0$, then we get a half-plane.)