Honors Math A
Practice problems, batch 2

1. Let \( f : [a, b] \to \mathbb{R} \) be a continuous function. Prove that \(|f|\) is continuous.

2. Let \( f : S \to T \) and \( g : T \to S \) be functions between sets \( S, T \) such that \( g \circ f : S \to S \) is bijective. Prove that \( f \) is injective.

3. Let \( f, g : [a, b] \to \mathbb{R} \) be continuous functions such that \( f(x) = g(x) \) for all \( x \in \mathbb{Q} \cap [a, b] \). Prove that \( f = g \).

4. Prove that if \( f \) is integrable, then so is \(|f|\).

5. Let \( a_n \) be a sequence such that \( a_0 > a_1 > a_2 > \cdots \) and such that \( \lim_{n \to \infty} a_n = 0 \). Prove that \( \sum_{n=0}^{\infty} (-1)^n a_n \) converges. (Hint: Let \( S_m = \sum_{n=0}^{2m} (-1)^n a_n \). Prove that \( S_i \geq S_{i+1} \) for all \( i = 1, 2, 3, \ldots \), and that the sequence \( S_i \) is bounded below.)

6. If \( f : [a, b] \to \mathbb{R} \) is a function such that \( f \) and \(|f|\) are integrable, prove that

\[
\left| \int_a^b f(x) \, dx \right| \leq \int_a^b |f(x)| \, dx.
\]