Honors Math A
Homework 2

Read the rest of the Introduction in Apostol (skipping starred sections if you wish). Then do the following problems. As before, hand in only the starred ones.

*1. (a) Prove that if $A$ and $B$ are sets, then their power sets satisfy $P(A) \cup P(B) \subseteq P(A \cup B)$. (b) Give an example, however, to show that equality need not hold in the above.

*2. Use the definition of ordered pair equality to prove that if $A, B, C$ are sets, then $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

3. If $f : S \rightarrow T$ and $g : T \rightarrow U$ are injective functions, prove that the composite $g \circ f$ is also injective.

*4. Suppose that $f : S \rightarrow T$ and $g : T \rightarrow S$ are functions such that $g \circ f = id : S \rightarrow S$. (In this case, we say that $g$ is a left inverse for $f$.) For each of the following, give a proof if true, or a counterexample if false. (a) $f$ is injective; (b) $f$ is surjective; (c) $g$ is injective; (d) $g$ is surjective.

Also do the following problems from Apostol. Apostol §I 3.3 (p. 19) 1 (do I.6-8), 3, 4*.
Apostol §I 3.5 (p. 21) 1 (do I.24*), 2, 5, 9, 10*. Apostol §I 4.4 (pp. 35-36) 1ab, 1c*, 4, 12.

“I had a feeling once about Mathematics - that I saw it all. Depth beyond depth was revealed to me - the Byss and Abyss. I saw - as one might see the transit of Venus or even the Lord Mayor’s Show - a quantity passing through infinity and changing its sign from plus to minus. I saw exactly why it happened and why the tergiversation was inevitable but it was after dinner and I let it go.”

-Winston Churchill