Choose six problems from the following list. If you attempt more than six, please choose which six problems should be graded. For true/false problems, give a proof or a counterexample as appropriate.

1. True/false: If \( f + g \) is integrable, then one of \( f \) or \( g \) is integrable as well.

2. Let \( S \) be a nonempty subset of the positive integers, and set \( S' = \{1 - \frac{1}{n} \mid n \in S\} \). Show that \( \sup(S') < 1 \) if and only if \( S \) is bounded above.

3. Let \( S, T \) be nonempty subsets of the real numbers, and suppose that
   i. \( \forall x \in S, y \in T, x \leq y \), and
   ii. \( \forall \epsilon > 0, \exists x \in S \) and \( y \in T \) such that \( y - x \leq \epsilon \).

   Prove that \( \sup(S) = \inf(T) \).

4. Prove that any polynomial is integrable on any closed interval. (Hint: Use integrability of monotone functions.)

5. True/false: If \( x, y \) are real numbers such that \( x + 1 \leq y \), there is an integer \( n \) such that \( x \leq n \leq y \).

6. Prove the triangle inequality: \( |x + y| \leq |x| + |y| \) for any real numbers \( x, y \).

7. If \( f : \mathbb{R} \to \mathbb{R} \) is an even function, show that \( F(x) = \int_0^x f(x)dx \) is an odd function.

8. Evaluate the limit \( \lim_{x \to 0} x \sin \frac{1}{x} \). (Hint: Use the "squeezing theorem.")

9. True/false: If \( S, T \) are nonempty bounded-above subsets of \( \mathbb{R} \) and \( f : S \to T \) is a surjective function such that \( f(x) < x \ \forall x \in S \), then \( \sup(T) < \sup(S) \).