Title: Polynomial dynamics and difference equations

Abstract:

We study dynamical systems of the form $g : \mathbb{A}_{\mathbb{C}}^n \to \mathbb{A}_{\mathbb{C}}^n$ of the form $(x_1, \dots , x_n) \mapsto (f_1(x_1), \dots , f_n(x_n))$ where each f_i is a polynomial over $\mathbb{C}$ of degree at least two. Using an old theorem of J.F. Ritt on compositional identities and results on the model theory of difference fields, we completely describe the periodic subvarieties of such dynamical systems. As corollaries we deduce a strong form of the dynamical Manin-Mumford conjecture for liftings of the Frobenius and Zhang's conjecture on the existence of algebraic points with dense forward orbits for such polynomial dynamical systems defined over number fields. [This is joint work with Alice Medvedev.]