TITLE: On the spectrum of spherical varieties over p-adic fields: Towards a 'relative' Langlands program?

ABSTRACT: Work of Jacquet and many others suggests that representations of a reductive group G (both in the local and in the automorphic setting) which are distinguished by a "large enough" subgroup H should be functorial lifts from some other group. I will try to formalize this concept with the help of a "dual group" attached to a spherical variety by Gaitsgory and Nadler. ($X := H\backslash G$ is called spherical if the Borel has an open orbit.) More precisely, I will discuss:

In the p-adic setting, with G split:

(1) Results on the unramified distinguished spectrum, spherical vectors and a spherical Plancherel formula.

(2) A conjecture describing the support of Plancherel measure in terms of the "dual group", and a Plancherel formula reducing this conjecture to the discrete spectrum (using a method of Bernstein).

(3) A theorem on finite multiplicities of the spectrum.

(4) Proof of a local non-degeneracy conjecture of Ichino and Ikeda.

In the automorphic setting:

(5) A reformulation of the Ichino-Ikeda conjecture on the Gross-Prasad period, which suggests a generalization to other spherical periods.

(6) Global-to-local motivation for the local conjecture.

Apart from (1), this is joint work with Akshay Venkatesh.