

Nonabelian Hodge theory in characteristic p

Simpson's nonabelian Hodge theory establishes an equivalence between the category of representations of the fundamental group of a compact Kähler manifold X and the category of Higgs bundles on X , subject to certain stability conditions. His correspondence is compatible with cohomology and provides an analog of the Hodge decomposition for cohomology with coefficients in a local system. Although his theory is not all algebraic, there is a similar theory in characteristic $p > 0$, in which the p -curvature plays the role of the Higgs field. Again there is an analog of the Hodge decomposition, generalizing the construction of Deligne and Illusie. These results are strong enough to give a “reduction mod p ” style proof of a theorem of Barranikov and Kontsevich about the cohomology of certain connections with irregular singularities—itsself a generalization of the Hodge decomposition. This is joint work with Vadim Vologodsky.