COMBINATORIAL NUMBER THEORY

MIDTERM EXAM (Monday, March 20, 2006 at 11:15 A.M.)

• 1.25 hour exam, closed book. Please clearly write out all answers to all 6 problems.

Problem 1: Let the continued fraction expansion of $\alpha, \beta$ be given by

$$\alpha = \langle a, 1, b, c, d, \ldots \rangle$$

$$\beta = \langle a, 1 + b, c, d, \ldots \rangle,$$

where it is presumed that both continue in exactly the same manner. There is an extremely simple equation relating $\alpha$ and $\beta$. Find this relation and show it is correct.

Problem 2: Let

$$\alpha = \langle 4, 1, 1, 4, 1, 1, 4, 1, 1, 1 \ldots \rangle = \langle 4, 1, 1, 1 \rangle.$$  

What is $\alpha$?

Problem 3: Explain why $\sum_{n=0}^{\infty} 10^{-n!}$ is a transcendental number.

Problem 4: Show that $3 + 2\sqrt{-5}$ is a prime in $\mathbb{Q}(\sqrt{-5})$.

Problem 5: Prove that if $\alpha$ is an integer in $\mathbb{Q}(\sqrt{d})$ and $N(\alpha)$ (the norm of $\alpha$) is a rational prime, then $\alpha$ is a prime in $\mathbb{Q}(\sqrt{d})$.

Problem 6: Define the Moebius function $\mu(n)$. Show that

$$\sum_{d|n} \mu(d) = \begin{cases} 1 & \text{if } n = 1, \\ 0 & \text{otherwise}. \end{cases}$$