

**Speaker:** Alexander Gamburd

**Title:** Arithmetic and Dynamics on Markoff-Hurwitz Varieties

**Abstract:** Markoff triples are integer solutions of the equation

$$x^2 + y^2 + z^2 = 3xyz$$

which arose in Markoff's spectacular and fundamental work (1879) on diophantine approximation and has been, henceforth, ubiquitous in a tremendous variety of different fields in mathematics and beyond. After reviewing some of these, we will discuss joint work with Bourgain and Sarnak on the connectedness of the set of solutions of the Markoff equation modulo primes under the action of the group generated by Vieta involutions, showing, in particular, that for almost all primes the induced graph is connected. Similar results for composite moduli enable us to establish certain new arithmetical properties of Markoff numbers, for instance the fact that almost all of them are composite.

We will also discuss recent joint work with Magee and Ronan on the asymptotic formula for integer points on Markoff-Hurwitz varieties

$$x_1^2 + x_2^2 + \cdots + x_n^2 = x_1 x_2 \cdots x_n,$$

giving an interpretation for the exponent of growth in terms of certain conformal measure on the projective space.