

Speaker: Vesselin Dimitrov

Title: New results in transcendence by arithmetic holonomy bounds

Abstract: We prove that the product of two logarithms $\log(1-1/n) \log(1-1/m)$ is irrational for all integer pairs n, m whose ratio n/m is sufficiently near to 1. This reports on a joint work in progress with Frank Calegari and Yunqing Tang, and is based on a new arithmetic upper bound on the $\mathbb{Q}(x)$ -dimensions of certain spaces of solutions of linear differential equations over $\mathbb{Q}(x)$. I will detail a proof of one of our arithmetic holonomicity theorems based on Bost's slopes method, indicate more briefly how the same circle of ideas also resolves a conjecture of Hall and Ruzsa on pseudopolynomials, explain the application to two logarithms, compare to the state-of-art in the Siegel–Bombieri arithmetic theory of special values of G -functions, and conclude by raising a specific stumbling block question about potentially going beyond two logarithms.