

# Laplace Transforms

	$t$ -domain $f(t) = \mathcal{L}^{-1}\{F(s)\}(t)$	$s$ -domain $F(s) = \mathcal{L}\{f(t)\}(s)$
delayed unit step	$u(t - a)$	$\frac{e^{-as}}{s}$
$n$ -th power	$t^n$	$\frac{n!}{s^{n+1}}$
exponential	$e^{at}$	$\frac{1}{s - a}$
sine	$\sin(bt)$	$\frac{b}{s^2 + b^2}$
cosine	$\cos(bt)$	$\frac{s}{s^2 + b^2}$
exponential sine	$e^{at} \sin(bt)$	$\frac{b}{(s - a)^2 + b^2}$
exponential cosine	$e^{at} \cos(bt)$	$\frac{s - a}{(s - a)^2 + b^2}$
definition	$f(t)$	$\int_0^\infty e^{-st} f(t) dt$
linearity	$af(t) + bg(t)$	$aF(s) + bG(s)$
time differentiation	$f'(t)$	$sF(s) - f(0)$
second time differentiation	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
general time differentiation	$f^{(n)}(t)$	$s^n F(s) - s^{n-1}f(0) - \dots - f^{(n-1)}(0)$
frequency differentiation	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
frequency shift	$e^{at} f(t)$	$F(s - a)$
time shift	$f(t - a)u(t - a)$	$e^{-as}F(s)$
time scaling	$f(at)$	$\frac{1}{ a }F\left(\frac{s}{a}\right)$
convolution	$(f * g)(t) := \int_0^t f(\tau)g(t - \tau) d\tau$	$F(s)G(s)$
frequency integration	$\frac{f(t)}{t}$	$\int_s^\infty F(\sigma) d\sigma$
periodic function	$f(t)$ with period $T$	$\frac{\int_0^T e^{-st} f(t) dt}{1 - e^{-Ts}}$

- $t \in [0, \infty)$  typically represents time.
- $s$  represents an “angular frequency”.
- $a$  and  $b$  are real numbers.
- $n$  is a nonnegative integer.