Problems 1

Due: Monday, 19 September 2022 before 17:00 EDT

P1.1 Let B and C be R-complexes. If the canonical morphism $\begin{bmatrix} \mathrm{id}^B & 0 \end{bmatrix} \colon B \oplus C \to B$ is an isomorphism, then prove that $C = 0$.

P1.2 For any two R-complexes B and C, demonstrate that there exists an canonical isomorphism $\varsigma^{B,C} \colon B \oplus C \to C \oplus B$. Moreover, for any two morphisms $\beta \colon B \to B'$ and $\gamma \colon C \to C'$ of R-complexes, prove that the diagram

$$\begin{array}{ccc} C \oplus B & \xleftarrow{\;\varsigma^{B,C}\;} & B \oplus C \\ {\scriptstyle \gamma \oplus \beta}\big\downarrow & & \big\downarrow{\scriptstyle \beta \oplus \gamma} \\ C' \oplus B' & \xleftarrow{\;\varsigma^{B',C'}\;} & B' \oplus C' \end{array}$$

commutes.

P1.3 Let $\psi \colon A \to B$ and $\varphi \colon B \to C$ be two morphisms. If $\varphi \psi$ is an isomorphism and φ is a monomorphism, then show that φ and ψ are both isomorphisms.

P1.4 Let $\varphi \colon B \to C$ be a morphism, let $\pi \colon B \times_C B \to B$ and $\pi' \colon B \times_C B \to B$ be the two canonical morphisms of the fibred product, and let $\delta \colon B \to B \times_C B$ denote the unique morphism arising the universal property of the fibred product that satisfies $\pi \delta = \mathrm{id}^B = \pi' \delta$. Prove that the following are equivalent:

(a) the morphism φ is a monomorphism,
(b) the morphism δ is an isomorphism,
(c) the morphism δ is an epimorphism,
(d) the morphisms π and π' are equal.