Problems 12
Due: Tuesday, 8 December 2020

1. In any category C, provide a direct prove for each of the following:
- (i) The composition of two epimorphisms is an epimorphism.
- (ii) If the composition of two morphisms is an epimorphism, then the second morphism is an epimorphism.
- (iii) Every isomorphism is both an epimorphism and a monomorphism.

2. A ***poset*** is a set P with a binary relation $\leqslant_P$ which is reflexive, transitive and antisymmetric. More precisely, for all $x, y, z \in P$, we have the following:
- (reflexivity) The relation $x \leqslant_P x$ holds.
- (transitivity) The relations $x \leqslant_P y$ and $y \leqslant_P z$ imply that $x \leqslant_P z$.
- (antisymmetry) The relations $x \leqslant_P y$ and $y \leqslant_P x$ imply that $x = y$.

A function $f : P \to Q$ between two posets is ***order-preserving*** if, for all $x, y \in P$ satisfying $x \leqslant_P y$, we have $f(x) \leqslant_Q f(y)$.
- (i) Show that the collection of posets together with order-preserving functions form a category.
- (ii) In the category of posets, give an example of a bijective morphism between non-isomorphic posets.

3. Let G be a group. The ***center*** of G, denoted $Z(G)$, is the set of elements in G that commute with every element in G. The ***commutator*** of two elements g and h in G is the element $[g, h] := g^{-1}h^{-1}gh$. The ***commutator subgroup*** G' of G is the subgroup generated by all the commutators of elements in G.
- (i) Show that the assignment $G \mapsto Z(G)$ does *not* give a functor from Grp to Ab.
- (ii) Show that $G \mapsto G/G'$ does give a functor from the category of groups to the category of abelian groups.