

# Problems 1

Due: Friday, 17 September 2021 before 17:00 EDT

Students registered in MATH 402 should submit solutions to 4 of the following problems. Students in MATH 802 should submit solutions to all 5.

1. Use *The On-Line Encyclopedia of Integer Sequences*, founded by N.J.A. Sloane and available at <https://oeis.org/>, to identify the following sequences:

(i) 1, 1, 3, 7, 19, 47, 130, 343, 951, 2615, 7318, 20491, ...

(ii) 10, 219, 4796, 105030, 2300104, 50371117, ...

(iii) 1, 1, 1, 3, 16, 125, 1296, 16807, 262144, 4782969, ...

2. Use *MathSciNet* and the *arXiv* (available at

<http://www.ams.org.proxy.queensu.ca/mathscinet/>, and

<https://arxiv.org>

respectively) to answer the following questions:

(i) Count the “Journal” publications with the phrase “Catalan numbers” or “Catalan number” in their title.

(ii) How many combinatorics preprints were added to the e-print archives in July 2021?

3. Let  $m$  be the arithmetic mean of a finite set of real numbers. Use the pigeonhole principle to show that there exists at least one number in the set that is less than or equal to  $m$ .

4. Consider a collection of 6 people. Prove that there are either three mutual acquaintances (all have met before) or three mutual strangers (none have ever met before).

5. For all nonnegative integer  $n$ , let  $F_n$  denote the  $n$ -th Fibonacci number.

(i) For all positive integers  $n$ , show that

$$\begin{bmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n .$$

(ii) For all positive integers  $n$ , prove that  $F_n^2 - F_{n+1}F_{n-1} = (-1)^{n-1}$ .

(iii) By diagonalizing the matrix in part (i), rederive the Binet formula.