

Problem Set #12

MATH 387 : 2015

Due: Thursday, 2 April 2015

1. Prove that any rigid motion is a product of at most three reflections.

Hint. Any rigid motion is completely determined by the images of any three non-collinear points.

2. The *hyperbolic sine* and *hyperbolic cosine* functions are defined as $\sinh(x) := \frac{1}{2}(e^x - e^{-x})$ and $\cosh(x) := \frac{1}{2}(e^x + e^{-x})$. Given a hyperbolic triangle with vertices A, B, C , side lengths a, b, c , and interior angles α, β, γ , the *hyperbolic law of cosines* asserts that

$$\cosh(a) = \cosh(b)\cosh(c) - \sinh(c)\sinh(b)\cos(\alpha).$$

(a) Verify that $\cosh^2(x) - \sinh^2(x) = 1$.

(b) Using the hyperbolic law of cosines, derive the *hyperbolic law of sines* which asserts that

$$\frac{\sinh(a)}{\sin(\alpha)} = \frac{\sinh(b)}{\sin(\beta)} = \frac{\sinh(c)}{\sin(\gamma)}.$$

Hint. Express the fraction $\frac{\sin^2(\alpha)}{\sinh^2(a)}$ as symmetric function in a, b, c .

3. What was your favourite result in the course? Provide a short paragraph explaining your answer.

