Problem Set #4

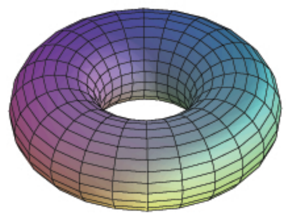
Due: 3 October 2008

1. Find an equation for the plane tangent to the surface $\vec{\boldsymbol{\sigma}} \colon \mathbb{R}^2 \to \mathbb{R}^3$ defined by

$$\vec{\boldsymbol{\sigma}}(s,t) := e^s \vec{\boldsymbol{\imath}} + t^2 e^{2s} \vec{\boldsymbol{\jmath}} + (2e^{-s} + t)\vec{\boldsymbol{k}}$$

at the point $(1, 4, 0)$.

2. A torus (doughnut) is constructed by rotating a small circle of radius a in a large circle of radius b about the origin. The small circle is in a (rotating) vertical plane though the origin and the large circle is in the xy-plane.



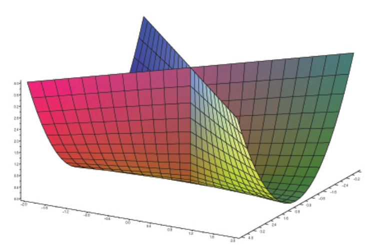
Parameterize the torus as follows:

(a) Parameterize the large circle.

(b) For a typical point on the large circle, find two unit vectors which are perpendicular to one another and in the plane of the small circle at that point. Use these vectors to parameterize the small circle relative to its center.

(c) Combine your answers in the first two parts to parameterize the torus.

3. The parametrized surface $\vec{\boldsymbol{\omega}} \colon \mathbb{R}^2 \to \mathbb{R}^3$ defined by $\vec{\boldsymbol{\omega}}(s,t) := st\vec{\boldsymbol{\imath}} + t\vec{\boldsymbol{\jmath}} + s^2\vec{\boldsymbol{k}}$ is called the ***Whitney umbrella***.



(a) Verify that this surface may also be described by the equation $y^2 z = x^2$ and inequality $z \geq 0$.

(b) Show that the parametrized surface $\vec{\boldsymbol{\omega}}$ nonsingular everywhere except the origin.

(c) Some points (x, y, z) of the surface do not correspond to a single parameter point (s, t). Which ones?

(d) Give an equation of the plane tangent to this surface at the point $(2, 1, 4)$.

(e) Show that at the point $(0, 0, 1)$ on the image of $\vec{\boldsymbol{\omega}}$ it's reasonable to conclude that there are *two* tangent planes. Give equations for them.