Problem Set #3

Due: 26 September 2007

1. **(a)** Show that the path $\vec{\boldsymbol{\gamma}} \colon \mathbb{R} \setminus \{\vec{\mathbf{0}}\} \to \mathbb{R}^3$ given by $\vec{\boldsymbol{\gamma}}(t) := e^{2t}\vec{\boldsymbol{\imath}} + \ln|t|\,\vec{\boldsymbol{\jmath}} + \frac{1}{t}\vec{\boldsymbol{k}}$ is a flow line of the vector field $\vec{\boldsymbol{F}} \colon \mathbb{R}^3 \to \mathbb{R}^3$ defined by $\vec{\boldsymbol{F}}(x,y,z) := 2x\vec{\boldsymbol{\imath}} + z\vec{\boldsymbol{\jmath}} - z^2\vec{\boldsymbol{k}}$.

(b) Find the flow lines of the vector field $\vec{\boldsymbol{G}} \colon \mathbb{R}^2 \to \mathbb{R}^2$ given by $\vec{\boldsymbol{G}}(x,y) := x\vec{\boldsymbol{\imath}} + 2y\vec{\boldsymbol{\jmath}}$.

2. For a vector field $\vec{\boldsymbol{F}} \colon X \subseteq \mathbb{R}^3 \to \mathbb{R}^3$, show that $\vec{\boldsymbol{\nabla}} \cdot (\vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{F}}) = 0$; in other words, the curl of a vector field is incompressible.

3. Find a vector field $\vec{\boldsymbol{F}} \colon \mathbb{R}^3 \to \mathbb{R}^3$ such that $\vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{F}} = 2\vec{\boldsymbol{\imath}} - 3\vec{\boldsymbol{\jmath}} + 4\vec{\boldsymbol{k}}$.

Hint. Try $\vec{\boldsymbol{F}} := \vec{\boldsymbol{v}} \times \vec{\boldsymbol{r}}$ where $\vec{\boldsymbol{v}} \in \mathbb{R}^3$ is a fixed vector and the vector field $\vec{\boldsymbol{r}} \colon \mathbb{R}^3 \to \mathbb{R}^3$ is given by $\vec{\boldsymbol{r}}(x,y,z) := x\vec{\boldsymbol{\imath}} + y\vec{\boldsymbol{\jmath}} + z\vec{\boldsymbol{k}}$.