Problem Set #1

Due: 12 September 2008

1. Let f be a differentiable function of one variable. If $w = f\left(\dfrac{x+y}{xy}\right)$, then show that

$$x^2 \frac{\partial w}{\partial x} - y^2 \frac{\partial w}{\partial y} = 0\,.$$

2. Consider the surface defined by the equation

$$x^3 z + x^2 y^2 + \sin(yz) = -3\,.$$

(a) Find an equation for the plane tangent to this surface at the point $(-1, 0, 3)$.

(b) Parametrize the line normal to this surface at the point $(-1, 0, 3)$.

Remark. For a surface $S \subset \mathbb{R}^3$, the ***normal line*** to S at $\vec{\boldsymbol{p}} \in S$ is the line that passes through the point $\vec{\boldsymbol{p}}$ and is perpendicular to S at $\vec{\boldsymbol{p}}$.

3. The surface $z = 3x^2 + \frac{1}{6}x^3 - \frac{1}{8}x^4 - 4y^2$ is intersected by the plane $2x - y = 1$. The resulting intersection is a curve on the surface.

(a) Parametrize this curve.

(b) Parametrize the line tangent to this curve at the point $\left(1, 1, -\frac{23}{24}\right)$.