Problems 05

Due: Friday, 10 February 2023 before 17:00 EST

P5.1. Demonstrate that the equation $x^6 + y^{12} = 703$ has no integer solutions.

P5.2. Determine whether the set $\mathbb{R} \cup \{\infty\}$ with addition and multiplication defined, for all x and y in $\mathbb{R} \cup \{\infty\}$, by

$$x \boxplus y := \min(x, y) \qquad \text{and} \qquad x \boxtimes y := x + y,$$

forms a commutative ring. If it is not, then list all of the defining properties that do hold and all those that fail to hold.

P5.3. Let $\mathbb{F}_4$ denote the subset of all (2×2)-matrices having the form

$$\begin{bmatrix} a & b \\ b & a+b \end{bmatrix}$$

where a and b are ring elements in the ring $\mathbb{Z}/\langle 2 \rangle$.

(i) Demonstrate that $\mathbb{F}_4$ is a subring of the ring formed by all (2×2)-matrices with entries in the ring $\mathbb{Z}/\langle 2 \rangle$.

(ii) Verify that $\mathbb{F}_4$ is a commutative ring.

(iii) Show that any nonzero element in $\mathbb{F}_4$ has a multiplicative inverse.