Problems 20

Due: Friday, 11 March 2022 before 17:00 EST

P20.1. Let V be the $\mathbb{C}$-vector space of trigonometric polynomials having degree at most 1. Consider the linear operator $J\colon V \to V$ defined, for all functions f in V, by $\big(J[f]\big)(x) := \int_0^{\pi} f(x-t)\, dt$. Show that J is diagonalizable and find an eigenbasis.

P20.2. Consider the linear operator $T\colon \mathbb{Q}[x]_{\leqslant 2} \to \mathbb{Q}[x]_{\leqslant 2}$ defined, for all polynomials f in $\mathbb{Q}[x]_{\leqslant 2}$, by

$$T[f] := \big(2\,f(0) - 4\,f(1)\big)\,1 - \big(f(-1) + f(1)\big)\,x + \big(f(-1) - 2\,f(0) + 6\,f(1)\big)\,x^2\,.$$

Determine whether T is diagonalizable.

P20.3. Find all scalars k such that the matrix $\mathbf{A} := \begin{bmatrix} 1 & -k & 2k \\ 1 & -k & 2k \\ 1 & -k & 2k \end{bmatrix}$ is diagonalizable.