Problems 11

Due: Friday, 26 November 2021 before 17:00 EDT

1. Find bases for the row space, the column space, and the kernel of

$$\mathbf{M} := \begin{bmatrix} 3 & 1 & -2 & 3 & 2 & -2 \\ -5 & -1 & 6 & -1 & -2 & -8 \\ -1 & 1 & 6 & 7 & 0 & -8 \\ -2 & 1 & 8 & 8 & -2 & 1 \end{bmatrix}.$$

2. Consider $\mathbf{A} := \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$. Find *all* subsets of columns of $\mathbf{A}$ that form a basis of its columns space.

3. Consider the matrix $\mathbf{B} := \begin{bmatrix} -3 & -2 & 4 \\ 1 & 3 & -3 \\ -2 & -3 & 4 \end{bmatrix}.$

(i) Show that the columns of the matrix $\mathbf{B}$ form a basis for $\mathbb{Q}^3$.

(ii) Calculate the matrix $\mathbf{C} := [(\vec{e}_1)_\mathbf{B} \ (\vec{e}_2)_\mathbf{B} \ (\vec{e}_3)_\mathbf{B}]$ where $(\vec{e}_j)_\mathbf{B}$ denotes the coordinate vector of $\vec{e}_j \in \mathbb{Q}^3$ relative to the columns of $\mathbf{B}$.

(iii) What is the relationship between $\mathbf{B}$ and $\mathbf{C}$?