Problems 10

Due: Friday, 19 November 2021 before 17:00 EDT

1. Consider the matrix $\mathbf{A} := \begin{bmatrix} -3 & -2 & 1 & -3 \\ 6 & 7 & -4 & 7 \\ 3 & 8 & -2 & 8 \\ -6 & -10 & 3 & -10 \end{bmatrix}$.

(i) Find an **LU**-factorization of **A**.

(ii) Using the **LU**-factorization, solve $\mathbf{A}\vec{\mathbf{x}} = \vec{\mathbf{b}}$ where $\vec{\mathbf{b}} := [3 \;\; -5 \;\; -7 \;\; 7]^{\mathsf{T}}$

2. An elementary matrix that differs from the identity matrix by interchanging a pair of consecutive rows is called an ***adjacent transposition***. Equivalently, an adjacent transposition is a matrix of the form $\mathbf{I} + \mathbf{E}_{j,j+1} + \mathbf{E}_{j+1,j} - \mathbf{E}_{j,j} - \mathbf{E}_{j+1,j+1}$ for some row index j.

(i) Express the permutation matrix $\mathbf{P} := \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ as a product of adjacent transpositions.

(ii) Prove that every permutation matrix is a product of adjacent transpositions.

3. For all $t \in \mathbb{C}$, find a $\mathbf{P}^{\mathsf{T}}\mathbf{LDU}$-factorization of the matrix $\mathbf{B} := \begin{bmatrix} 3t & -9t+2 & 8t+1 & 3t^2-7 \\ -3 & 8 & -t-6 & -3t+2 \\ 3 & -9 & 6 & 3t \\ -3 & -t+9 & -t^2-9 & -t+11 \end{bmatrix}$.