PROBLEM SET 2 (INTRODUCTION TO MATLAB)
Due: Monday, February 25, 2013

Problem 1 [4 points]:
Find the absolute error, relative error, and decimal precision (number of significant decimal digits) for the following objects \( f \) and their approximations \( \tilde{f} \).

a. \( f = \pi, \tilde{f} = 3.14 \)

b. \( f = \pi, \tilde{f} = \frac{22}{7} \)

c. \( f = \log (n!), \tilde{f} = n \log(n) - n \) for \( n = 5, 10, 100 \) (This is Stirling’s approximation)

d. \( f = e^3, \tilde{f} = T_n(3) \), where \( T_n \) is the n-th Taylor Polynomial approximation to \( e^x \) expanded around \( x = 0 \). Consider \( n = 1, 2, 3 \). What value of \( n \) is required for this approximation to be good to 6 digits of decimal precision?

Problem 2 [6 points]:
Suppose you are given the values of \( f \) and \( f' \) at points \( x_0 + h \) and \( x_0 - h \) and you wish to approximate \( f'(x_0) \). Find coefficients \( \alpha \) and \( \beta \) that make the following approximation accurate to \( O(h^2) \):

\[
f'(x_0) \approx \frac{f'(x_0 + h) + f'(x_0 - h)}{2} + \beta \frac{f(x_0 + h) - f(x_0 - h)}{2h}
\]

Compute the coefficients by combining the Taylor series expansions of \( f(x) \) and \( f'(x) \) about the point \( x_0 \):

\[
f(x) = f(x_0) + (x - x_0)f'(x_0) + \frac{(x - x_0)^2}{2!} f''(x_0) + \frac{(x - x_0)^3}{3!} f'''(x_0) + \frac{(x - x_0)^4}{4!} f^{(4)}(x_0) + \frac{(x - x_0)^5}{5!} f^{(5)}(c_1)
\]

\[
f'(x) = f'(x_0) + (x - x_0)f''(x_0) + \frac{(x - x_0)^2}{2!} f'''(x_0) + \frac{(x - x_0)^3}{3!} f^{(4)}(x_0) + \frac{(x - x_0)^4}{4!} f^{(5)}(c_2)
\]

[Hint: Combine the Taylor expansions into \( f(x_0 + h) - f(x_0 - h) \) and \( f'(x_0 + h) - f'(x_0 - h) \) and then combine these two to cancel the leading order error term (in this case \( O(h^2) \)).]