Problem 1: (10 Points)
Starting with the graph of \( y = e^x \), write the equation of the graph that results from
a. Shifting 3 units downward. [Answer: \( y = e^x - 3 \)]
b. Shifting 2 units to the right. [Answer: \( y = e^{x-2} \)]
c. Reflecting about the \( y \)-axis. [Answer: \( y = e^{-x} \)]

Problem 2: (10 Points)
Let \( f(x) = x^2 - 8 \), and \( g(x) = \frac{1}{1-x} \). Find the composition function \( g \circ f \), and its domain.
Answer:
\[
(g \circ f)(x) = \frac{1}{\sqrt{f(x)} - 1} = \frac{1}{\sqrt{x^2 - 8} - 1} = \frac{1}{\sqrt{x^2 - 9}}
\]

\( \text{Dom}(f) = \mathbb{R} \) (as \( f \) is a polynomial). \( \text{Dom}(g) = \{ x \in \mathbb{R} : x - 1 > 0 \} = \{ x \in \mathbb{R} : x > 1 \} \). Hence:
\( \text{Dom}(g \circ f) = \{ x \in \text{Dom}(f) : f(x) \in \text{Dom}(g) \} = \{ x \in \mathbb{R} : x^2 - 8 > 1 \} = \{ x \in \mathbb{R} : |x| > 3 \} = (-\infty, -3) \cup (3, +\infty) \).

Problem 3: (10 Points)
Find the exact value of the following expressions:
\( a. \ \log_2 80 - \log_2 10 \) \( \text{[Answer: } \log_2 (80/10) = \log_2 8 = \log_2 2^3 = 3 \] \)
\( b. \ \ln e^4 - \log_{10} 10 \) \( \text{[Answer: } 4 - 1 = 3 \] \)

Problem 4: (10 Points)
Explain in (your) words what is meant by the equation
\[
\lim_{x \to 2} f(x) = 5
\]
Is it possible for this statement to be true and yet \( f(2) = 3 \) ? Explain.
Answer:
By taking x to be sufficiently close to 2 (x ≠ 2), we can make the values of f(x) arbitrarily close to 5.
It is possible for this statement to be true and yet \( f(2) = 3 \); in fact, the function f does not need to be defined at x = 2 for the limit to exist.

**Problem 5: (10 Points)**
Evaluate
\[
\lim_{x \to 4} \frac{|x - 4|}{x - 4}
\]

if it exists. If the limit does not exist, explain why.

**Answer:**
\[
\lim_{x \to 4} \frac{|x - 4|}{x - 4} = \lim_{x \to 4} \frac{-(x - 4)}{x - 4} = \lim_{x \to 4} -1 = -1;
\]
\[
\lim_{x \to 4} \frac{|x - 4|}{x - 4} = \lim_{x \to 4} \frac{(x - 4)}{x - 4} = \lim_{x \to 4} +1 = +1;
\]

Since the two limits are different, we have that the \( \lim_{x \to 4} \frac{|x - 4|}{x - 4} \) does not exist.

**Problem 6: (10 Points)**
Evaluate
\[
\lim_{x \to 0} \frac{3x^2 - x - 2}{x^2 - 4}
\]

if it exists. If the limit does not exist, explain why.

**Answer:**
Divide the numerator and denominator by the highest power in the denominator (in this case \( x^2 \)); then we have:
\[
\lim_{x \to 0} \frac{3x^2 - x - 2}{x^2 - 4} = \lim_{x \to 0} \frac{3 - (1/x) - (2/x^2)}{1 - (4/x^2)} = 3
\]

**Problem 7: (10 Points)**
Show that \( \lim_{x \to 0} x^2 \sin \frac{1}{x} = 0 \) (Hint: Use the Squeeze Theorem)
Answer:

For all \( x \neq 0 \), \(-1 \leq \sin \frac{1}{x} \leq 1 \). Hence for all \( x \neq 0 \), \(-x^2 \leq x^2 \sin \frac{1}{x} \leq x^2 \).

We know that: \( \lim_{x \to 0^-} x^2 = \lim_{x \to 0^+} x^2 = 0 \).

Hence by the Squeeze Theorem, we can conclude that \( \lim_{x \to 0} x^2 \sin \frac{1}{x} = 0 \).

Problem 8: (10 Points)

Use the Intermediate Value Theorem to show that there is a root of the equation

\[ x^3 + x^2 + 2 = 0 \]

in the interval \((-2, -1)\).

Answer:

For all \( x \in \mathbb{R} \), let \( f(x) = x^3 + x^2 + 2 \). Note that \( f \) is a polynomial function, hence it is continuous everywhere; in particular \( f \) is continuous on \([-2, -1]\). Also, \( f(-2) = -2 \) and \( f(-1) = 2 \). Then by the Intermediate Value Theorem, there is a \( c \in (-2, -1) \) such that \( f(c) = 0 \).

Problem 9: (20 Points)

Let

\[
   f(x) = \begin{cases} 
   3x - 4 & \text{if } x > 2 \\
   1 & \text{if } x = 2 \\
   |x| & \text{if } x < 2
   \end{cases}
\]

1. Sketch the graph of \( f \) and find its domain, and range.

2. Find \( \lim_{x \to 2^-} f(x) \) and \( \lim_{x \to 2^+} f(x) \).

3. Does \( \lim_{x \to 2} f(x) \) exist?

4. On which interval(s) is \( f \) continuous?

Answer:

1. \( \text{Dom}(f) = \mathbb{R} \); \( \text{Range}(f) = [0, +\infty) \)

2. \( \lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} |x| = \lim_{x \to 2^-} x = 2 \); \( \lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} 3x - 4 = 3(2) - 4 = 2 \);

3. Yes, because \( \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = 2 \)
4. If \( a < 2 \), then \( f \) is continuous at \( a \) as \( f(x) = |x| \) is a continuous function. If \( a > 2 \), then \( f \) is continuous at \( a \) as \( f(x) = 3x - 4 \) is a polynomial function. If \( a = 2 \), then \( \lim_{x \to 2} f(x) = 2 \neq f(2) = 1 \). Hence \( f \) is not continuous at \( a = 2 \). Therefore \( f \) is continuous on \(( -\infty, 2) \cup (2, +\infty) \).