**Summary**

The task of discounting flows under collaterals from a pool of bootstrapped curve, is the same problem as computing a FX given a list of FXs (via different crosses determined on the fly).

**Details**

Referring to [MFAT], or [1], it can be shown that the discount factor for a flow at time \( t \) in currency \( i \) while collateral is in currency \( k \) is given by

\[
e^{-\left( r^{(i)}(t) + c^{(k)}(t) \right) T}
\]

where notations used is the naturally simplified ones from the following:
- \( r^{(i)}(t) \) is the risk-free continuous compounding zero rate for currency \( i \) at time \( t \),
- \( c^{(k)}(t) \) be the continuous compounding collateral return rate for at time \( t \).

Let \( {k \atop i}^T = e^{-\left( r^{(i)} - r^{(k)} + c^{(k)} \right) T} \) the discount factor for currency \( i \) with collateral \( k \) at time \( T \). We will omit \( T \) when it is understood.

**Proposition 1**: for any currencies \( x, y, z, w \) we have

\[
{w \atop x} {y \atop z} = {w \atop y} {x \atop z}
\]

The proof is straightforward from the discount factor expression.

Hence we see that:

**Inversion formula**: \( {w \atop x} {x \atop w} = {w \atop w} {x \atop x} \)

**Transitive formula**: \( {w \atop x} {x \atop y} = {w \atop y} {x \atop y} \)

Consider the task of computing the discount factor \( {k \atop i} \) among a given collection of currencies. Suppose also that for any currency \( x \), \( {x \atop x} \) is known. That is, each domestic collateral return rate is known in its respective domestic market (which seems to be not a harsh assumption). Then, the inversion formula says that if \( {w \atop x} \) is known, \( {w \atop w} \) is also known. The transitive formula says, modulo the term \( {x \atop x} \), the DF behaves like an FX.

Following the tradition of FX, if we put an arrow from \( x \) to \( y \) whenever \( {x \atop y} \) is given / known, a directed graph is formed. Let this graph be denoted by \( G \).

**Proposition 2**: \( {k \atop i} \) can be computed if and only if \( i \) and \( k \) belong to the same connected component in \( G \)

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\(^1\) Formulas named by my colleague Arnaud Lederc
Proof: the “if” part is straightforward from above. Indeed one can trace out the chain of DF involved with any directed path connecting $i$ and $k$.

The “only if” part can be traced back by the form of the DF expression by omitting the $c$ term. QED

**Conclusion:** the task of discounting flows under collaterals from a pool of bootstrapped curve, is the same problem as computing a FX given a list of FXs (via different crosses determined on the fly)

### Reference


### Contact

**Tat Sang Fung, PhD**
Adjunct Assistant Professor, Mathematics Department, Columbia University
Head of Financial Engineering, Senior Manager, Product Management (Misys Summit)

Email: tat.fung@misys.com
Academic Email: fts@math.columbia.edu

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