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Overview

This is a tentative course structure outline and is subject to change
Document updated April 4, 2023

The course

This course is an introduction to Linear Algebra. Students will learn to apply various conceptual and
computational techniques useful to tackle problems in linear systems, finite or infinite dimensional. We
will start with “solving $n$ linear equations in $m$ unknowns” which naturally leads to the notion of matrices.
We will then study matrix properties and its many roles in vector spaces. Examples and assignments are
drawn from applications readily found in many areas like physics, chemistry, finance.

This course is designed to be both theoretical and practical. Students are challenged in the following
aspects:

- Theoretical
  - Theorems, proofs (and how to write a coherent proof)
  - Geometric intuitions
  - Imagining algorithms, process and its outcome

- Practical
  - Concrete calculations, results interpretations
  - Applying algorithms to numerical examples

- Technology (if time and classroom setting permits)
  - Experimenting using Excel with a little bit VBA coding and/or Python

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This course is particularly interesting for those who want to acquire an understanding of the abstract theory as well its practical applications in different areas with a balance finetuned based on the class backgrounds compositions

**PRE-REQUISITES**

There is no pre-requisite on the critical path. Your prior experience with additions, subtractions, multiplications and divisions plus your imagination should make this class accessible. In fact, the more open/imaginative you are with the meaning of these operations and the objects behind, the more fun you will find this journey. Some students may find this class enlightening while pondering deeply things they have been relying on but seldom find the time to question (e.g. does \( ab = ba \) always hold? What exactly does “solve the equation \( ax = b \)” mean? If \( a^2 = 0 \) does it always imply \( a = 0 \)?) and find a need to “unlearn”/re-examine certain prior lessons and as a result attain a much deeper understanding in the nature of mathematics. In this sense, your open-mindedness is a pre-requisite.

Basic working knowledge with complex numbers is required. For example, we will use the fundamental theorem of algebra which states that every non-constant single-variable polynomial with complex coefficients has at least one complex root. In addition, it would be helpful to be familiar with vectors in \( \mathbb{R}^2 \) and \( \mathbb{R}^3 \). Some examples to be covered in class may require prior understanding of the following areas (nice-to-know)

**Mathematics:** Calculus (derivative of a function), elementary analysis (e.g. limits of a sequence), probability theory (e.g. distribution) and statistics (mean, variance)

**Computing Skills:** Implementation of algorithm involving calculations with a good working knowledge of Microsoft Excel. Students who have never done a programming project or have never created serious spreadsheets may find this course a good introduction to such tools

**WHO IS IT FOR?**

If you intend to learn serious science subjects (e.g. Physics, Chemistry, Social Sciences, Mathematics) Linear Algebra is a language which you need to master. It also can serve as a good introductory course to abstract algebra and at the same time sharpen your geometric intuition when solving problems

**TEXTBOOKS**

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There will be homework problems assigned based on the textbook above. It would be a good idea to buy/borrow a copy so you have ready access. Columbia University Mathematical Library should have a copy on the reserve shelf.

**COURSE CONTENTS**

We intend to cover Chapter 1 to 8 of the textbook with some omissions of non-essential sections, and adding some topics if time permits.

Topics to cover are: systems of linear equations, Gaussian elimination, matrices, reduced row echelon form, matrix algebra, linear transformations and its geometric meanings, linear spaces and its subspaces,
dimension, isomorphism, kernel, image, basis, orthogonality, Gram-Schmidt algorithm, QR factorization, determinants, eigenvalues, eigenvectors, symmetric matrices, Quadratic forms

**REQUIRED WORK**

Students are required to complete homework assignments. They concern both theoretical and practical aspects of the topics covered in class. For the theoretical section students are required to perform mathematical calculations and proofs. For the practical section students are required to perform tasks and experiments. Having access to a laptop helps.

There will be midterm and final exams. Classroom participation and other factors will also contribute to the final grade. The exact proportions will be determined when class begins.

**GRADING**

We will determine the percentage contribution of homework, midterm, final exam, and classroom participation towards the final grade when class begins.

**GRADING POLICY**

**INTEGRITY**

All solutions to the homework, test and exams (take home or otherwise) should be your work. Academic common sense should provide a good guideline and if you are in doubt please consult the instructor. A substantiated violation of the code of integrity and/or academic dishonesty (homework copying for example) may result in serious academic disciplinary action (including but not limited to a failing grade of this course).

**LATE POLICY**

Late assignment receives no points. If you still want to hand it in, it should be given directly to the TA.

Late or omitted assignments due to exceptional circumstances (e.g. serious illness with doctor’s note or emergency) would be handled on a case-by-case basis.

**ABOUT THE INSTRUCTOR**


Tat Sang Fung has been teaching graduate level classes at Columbia since Spring 2006. He can be reached at fts@math.columbia.edu

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