Columbia University
MATH GR5030 Spring 2018
Numerical Methods in Finance
Course Overview

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OVERVIEW

This is a tentative course structure outline and is subject to change
Document updated Nov 2, 2017

THE COURSE

This class was coded G6071 before year 2017, and starting Spring 2017 it is GR5030.

This course introduces and applies various numerical and computational techniques useful to tackle problems in mathematical finance. Among them are different interpolation methods and their consequences during hedge, root solving techniques and their properties. The focus of this course is the pricing of derivative securities. PDE (partial differential equation) approach is discussed and their stability analyzed. Monte Carlo methods are introduced with various variance reduction techniques and their theoretical aspects are studied. We will also include applications to credit derivatives, and other fashionable topics if time permits.

The course is designed to be both theoretical and practical. In the class we will deal with theoretical aspects of the numerical techniques (what works, and when it does not work, what is popular in the industry and why) using tools from pure and/or applied mathematics, with spreadsheet experimentations. Students are expected to do the same for homework assignments: some problems will be theoretical, while for the practical section spreadsheets are to be made. In this course students are challenged in both areas: Theoretical (theorems, calculations, proofs) and Practical (making spreadsheets that are working, easy to use and understand).

Our Emphasis:

• Theory
• Understanding
• Experiment
• Communications
Hence the course would be particularly interesting for those who want to consider quantitative positions in banking and insurance industries.

**PRE-REQUISITES**

**Quantitative Finance:** Familiarity with the materials covered in W4071 Mathematics: Introduction to Math Finance. Roughly it means the first 18 chapters of Hull (Book 5 in the Textbooks section)

**Mathematics:** Strong background of linear algebra (matrices, especially eigenvalues and eigenvectors), Calculus (Taylor’s expansion, integration and multivariable calculus), elementary analysis (e.g. limits), probability theory (distribution, conditional probabilities etc), stochastic calculus (e.g. Ito’s lemma) and statistics (mean, variance, moment generating functions) are necessary.

**Computing Skills:** Ability to implement algorithm involving complex calculations. Must have a good working knowledge of Microsoft Excel to complete the homework assignments, tests and exams. Students who have never done a programming project or have never created serious spreadsheets may experience extra challenges.

**WHO IS IT FOR?**

If you want to enhance your experience in an area that draw heavily on your mathematics, quantitative finance and implementation skills, this course may be for you.

**TEXTBOOKS**

<table>
<thead>
<tr>
<th>Name</th>
<th>Authors</th>
<th>Details</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 Excel 2007 VBA Programmer's Reference</td>
<td>Paul T. Kimmel, Stephen Bullen, John Green, Rob Bovey, Robert Rosenberg, Brian Patterson</td>
<td>Wrox (March 26, 2007) ISBN 0470046430</td>
<td>Recommended / Required for Excel homework</td>
</tr>
<tr>
<td>6 Interest Rate Models - Theory and Practice: With Smile, Inflation and Credit</td>
<td>Damiano Brigo (Author), Fabio Mercurio</td>
<td>Springer; 2nd ed. 2006. Corr. 3rd printing edition (September 26, 2007);</td>
<td>Recommended reference</td>
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COURSE CONTENTS

We intend to cover the following topics:

- Interpolations and root solving techniques that are used frequently in Quantitative Finance
- Pricing derivatives by the Partial Differential Equation approach (Explicit, Implicit, Crank-Nicolson Method and their stability analysis. American Option pricing as a free boundary problem)
- Pricing derivatives by the Monte Carlo Method (path generation, variance reduction techniques)

REQUIRED WORK

Students are required to complete homework assignments. They concern both theoretical and practical aspects of the topics covered in class. For the theoretical section students are required to perform mathematical calculations and proofs. For the practical section students are required to perform tasks and experiments using Microsoft Excel.

There will be interim tests and final exams. Classroom participation and other factors will also contribute to the final grade. The exact proportions will be determined later when the semester begins.

In recent spring semesters we have 4 (or 5) homework assignments, a midterm and a Final exam. Student can expect approximately an assignment due every two weeks.

GRADING

Although we want to decide the exact final grade determination schema at the beginning of the semester, it is arguably not the best practice, for it takes into no consideration of the actual characteristics of the students who are taking the class this year. As a result, below is a plan of how it might work. We will finalize the proportions in Feb or early March.

The following may sever as a starting point:

<table>
<thead>
<tr>
<th>Comments</th>
<th>Percentage</th>
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<tr>
<td>Home works Practice and Theory</td>
<td>30.00%</td>
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<td>Mid term Theory</td>
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<tr>
<td>Final Exam Theory</td>
<td>40.00%</td>
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<tr>
<td>Others (e.g. Class participation)</td>
<td>5.00%</td>
</tr>
<tr>
<td>Total</td>
<td>100.00%</td>
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GRADING POLICY

INTEGRITY

All solutions to the homework, test and exams (take home or otherwise) should be your work. Academic common sense should provide a good guideline and if you are in doubt please consult the instructor. A substantiated violation of the code of integrity and/or academic dishonesty (homework copying for
example) may result in serious academic disciplinary action (including but not limited to a failing Grade of this course)

**Late Policy**

Homework due date and time in general follows the following convention:

- For written solutions, they are expected to be collected at the beginning of the class
- For spreadsheet solutions, they are expected to be emailed to the TA on or before the beginning of the relevant class

Late assignment receives no points. If you still want to hand it in, it should be given directly to the TA. Late or omitted assignments due to exceptional circumstances (e.g. serious illness with doctor’s note or emergency) would be handled on a case-by-case basis.

**About the Instructor**

Tat Sang Fung is a Senior Manager in Misys. He joined Summit in 1996 where he specializes in financial engineering and quantitative techniques. He coauthored the article "BGM numeraire alignment at will" published in Risk International, 2004. Tat Sang Fung holds a Ph.D. in Mathematics from Columbia University in the City of New York. In the past he has taught Differential Equations and Numerical Methods, Advanced Calculus, Basic Mathematics, College Algebra and Analytic Geometry.

Tat Sang Fung has been teaching this class at Columbia since 2006 spring semester.

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